Physics of Materials

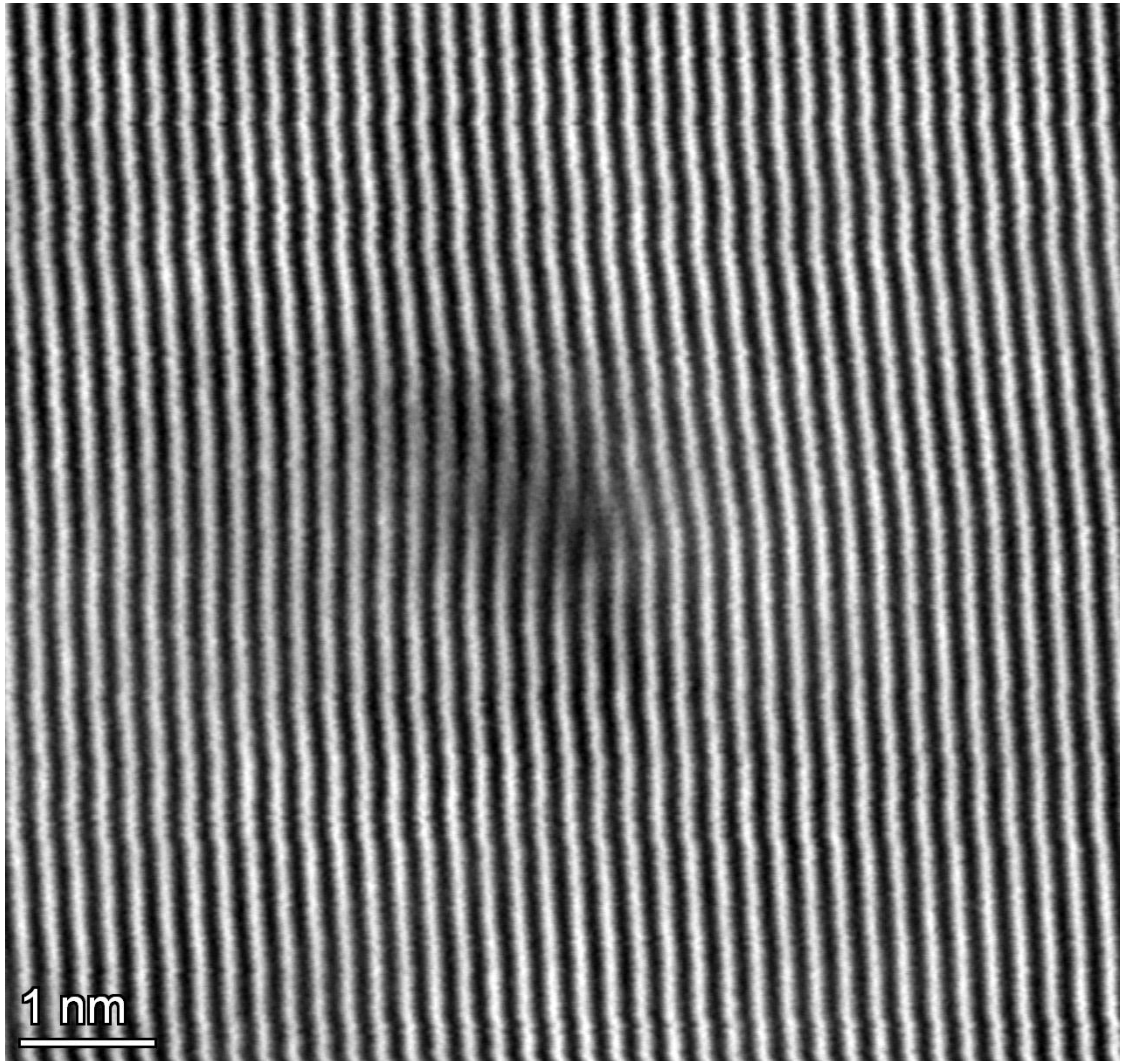
Dr. Thomas LaGrange

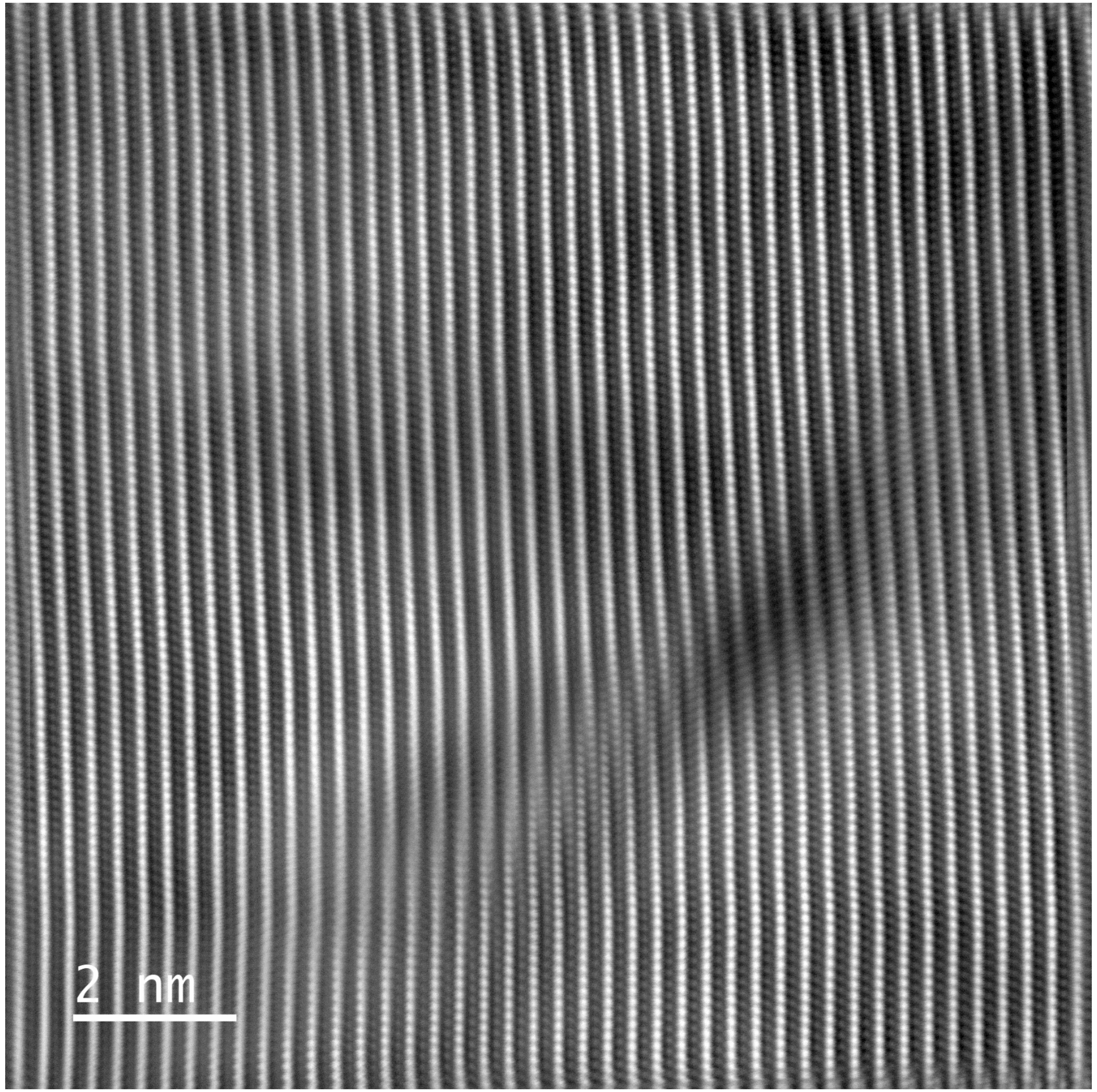
Chapter 6: Plastic Deformation:
Introduction to the Dislcolation model

Masters Course PHYS-307

Fall 2025

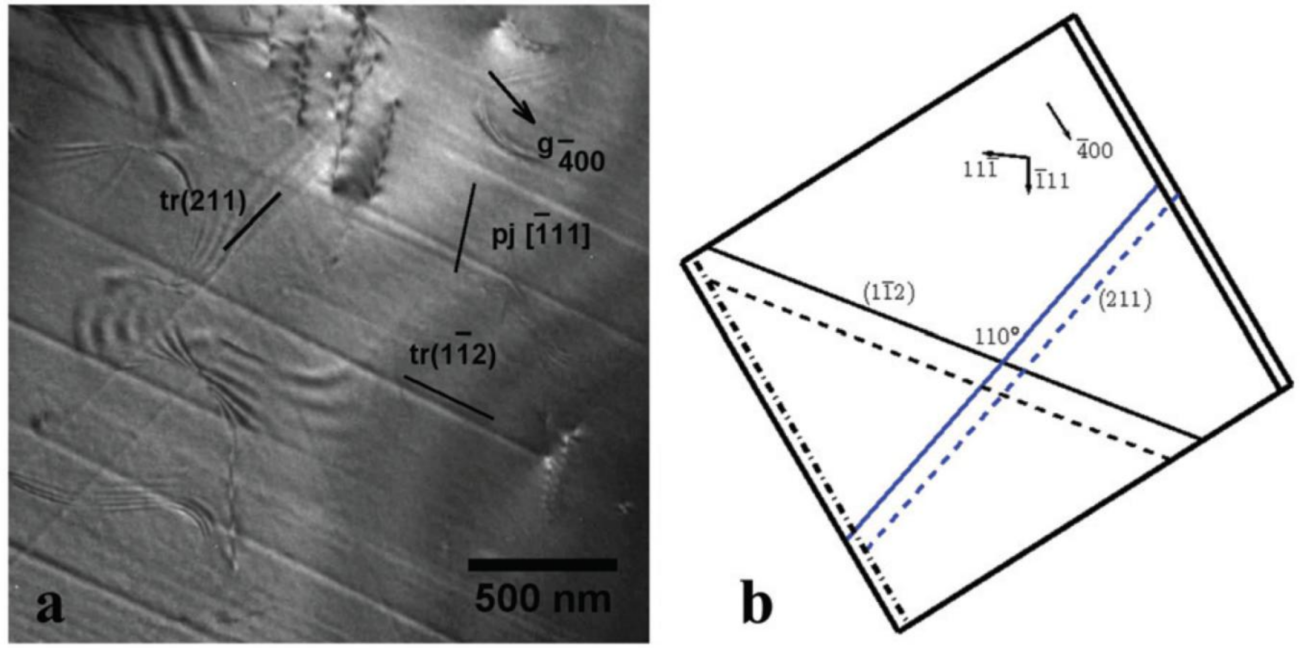
Dislocations

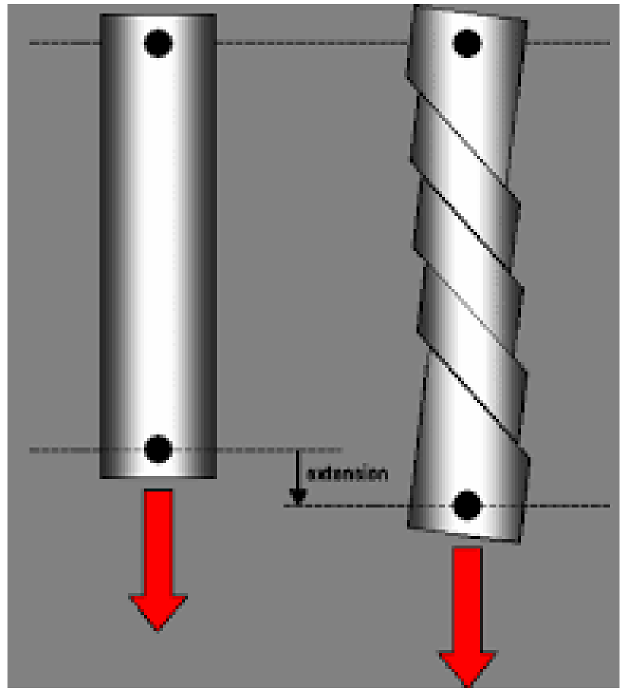


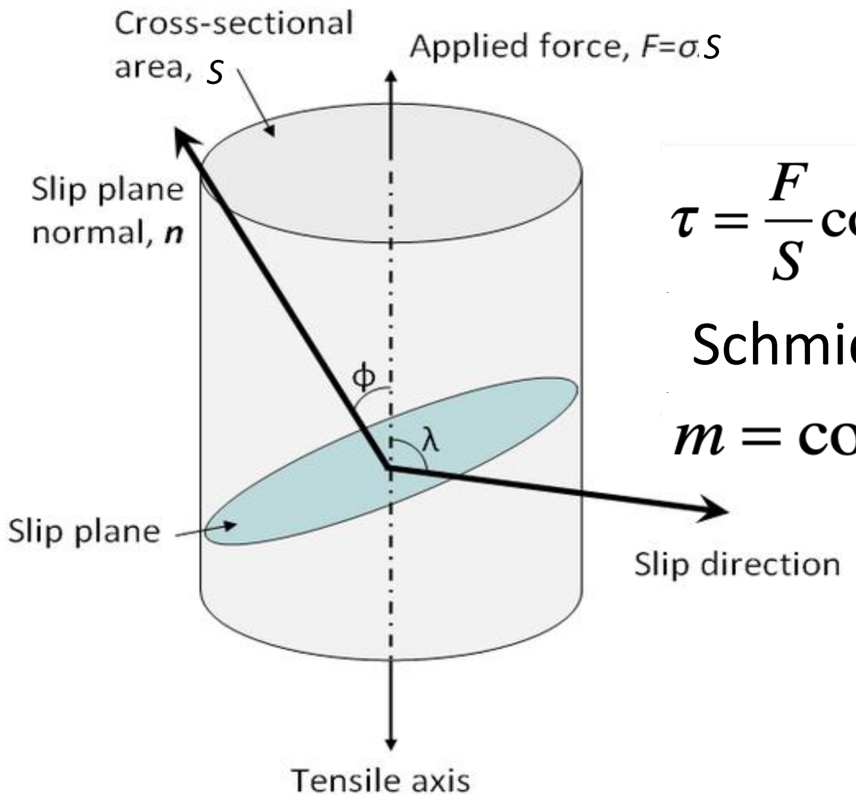


Dislocations were first observed in Single Crystals

Slip lines in a single-crystal



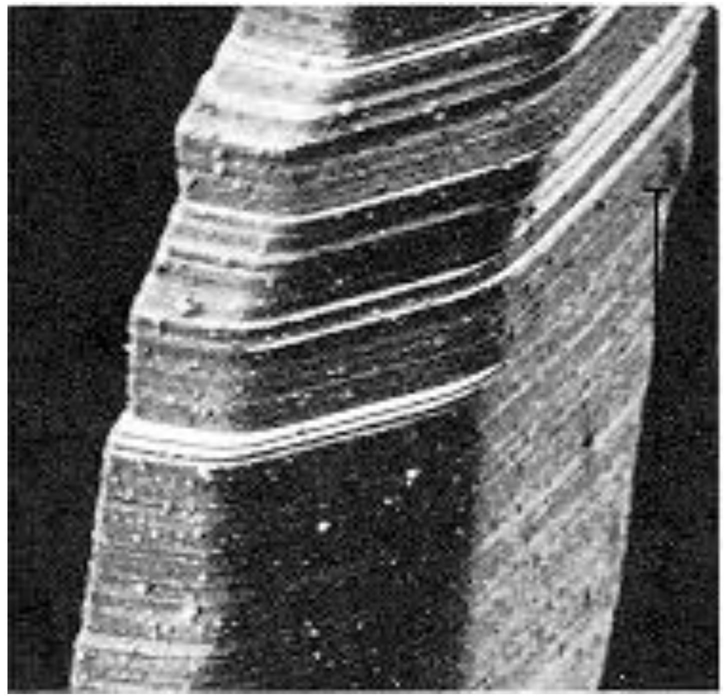




$$\tau = \frac{F}{S}\cos\phi\cos\lambda$$

Schmid factor

$$m = \cos\phi\cos\lambda$$



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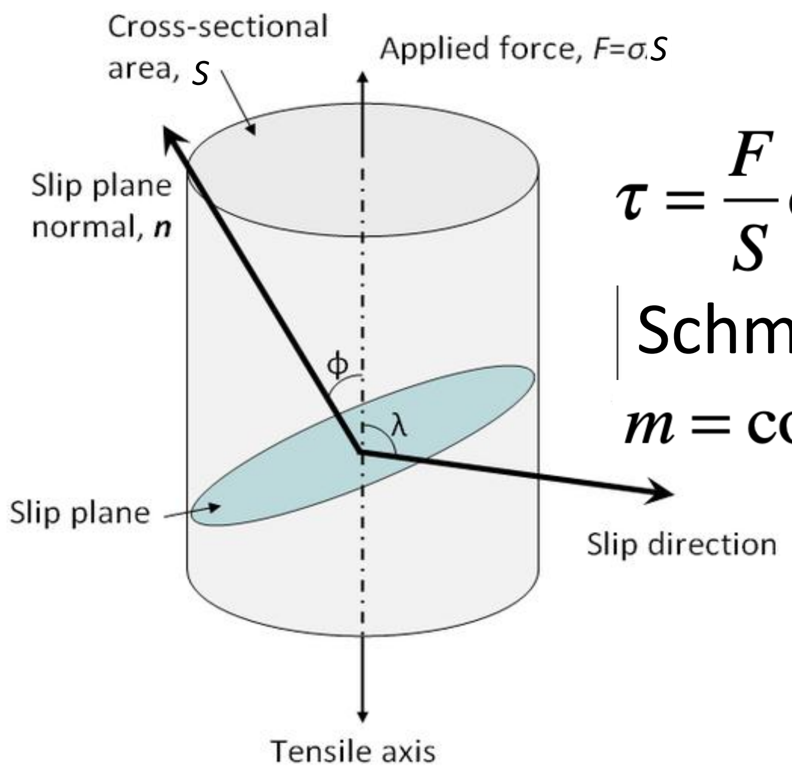
On the track of dislocations



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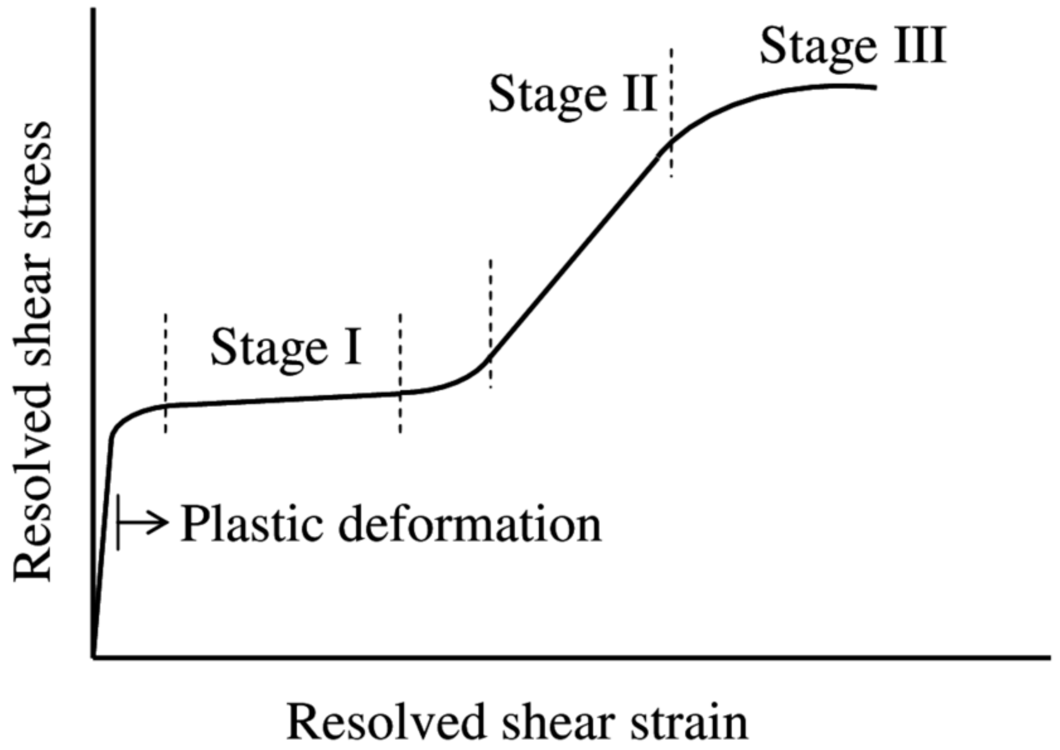
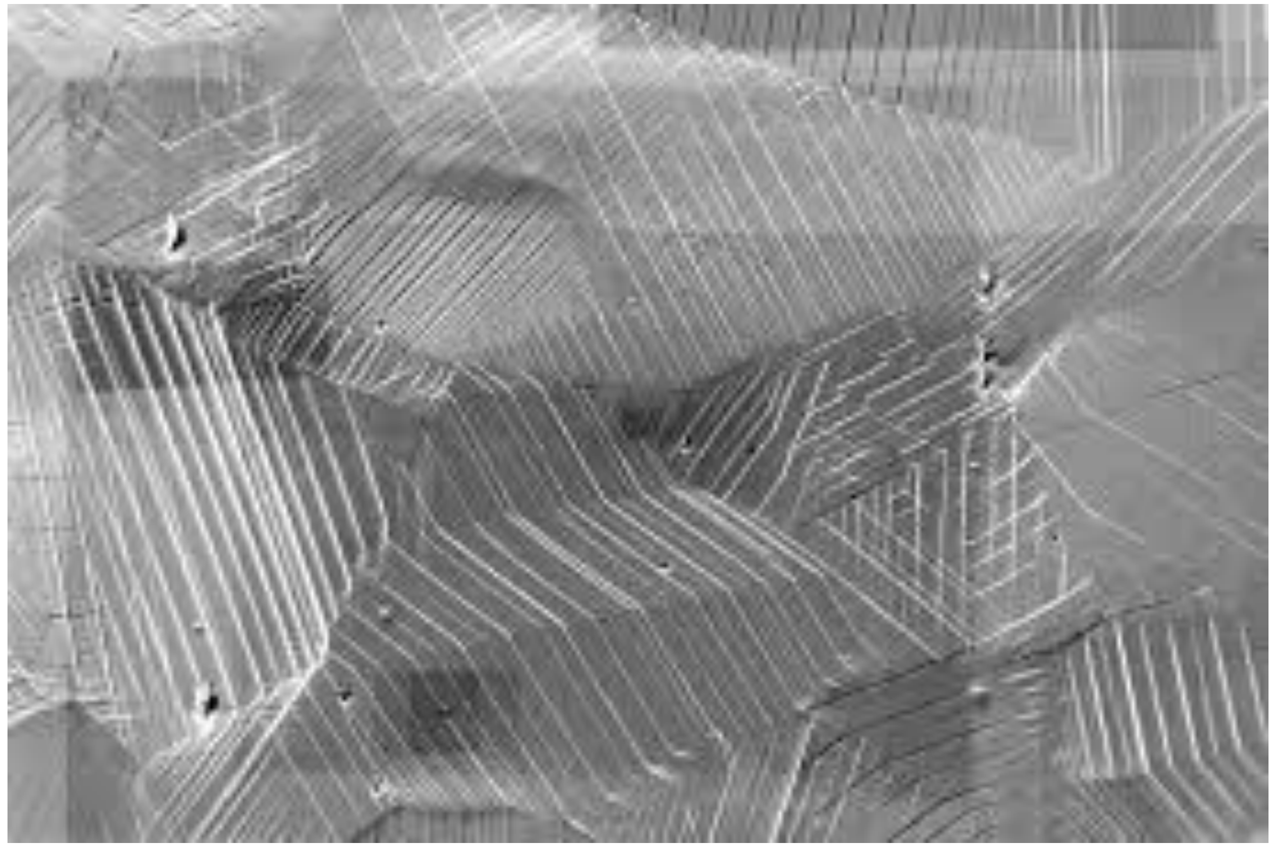


Slip lines in a polycrystal



- Stage 1: easy glide, dislocation motion
- Stage 2: Hardening from dislocations (with the highest Schmid factor) pile-ups
- Stage 3: hardening from activation of other slip systems and dislocations cutting through each other

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Mechanical annealing

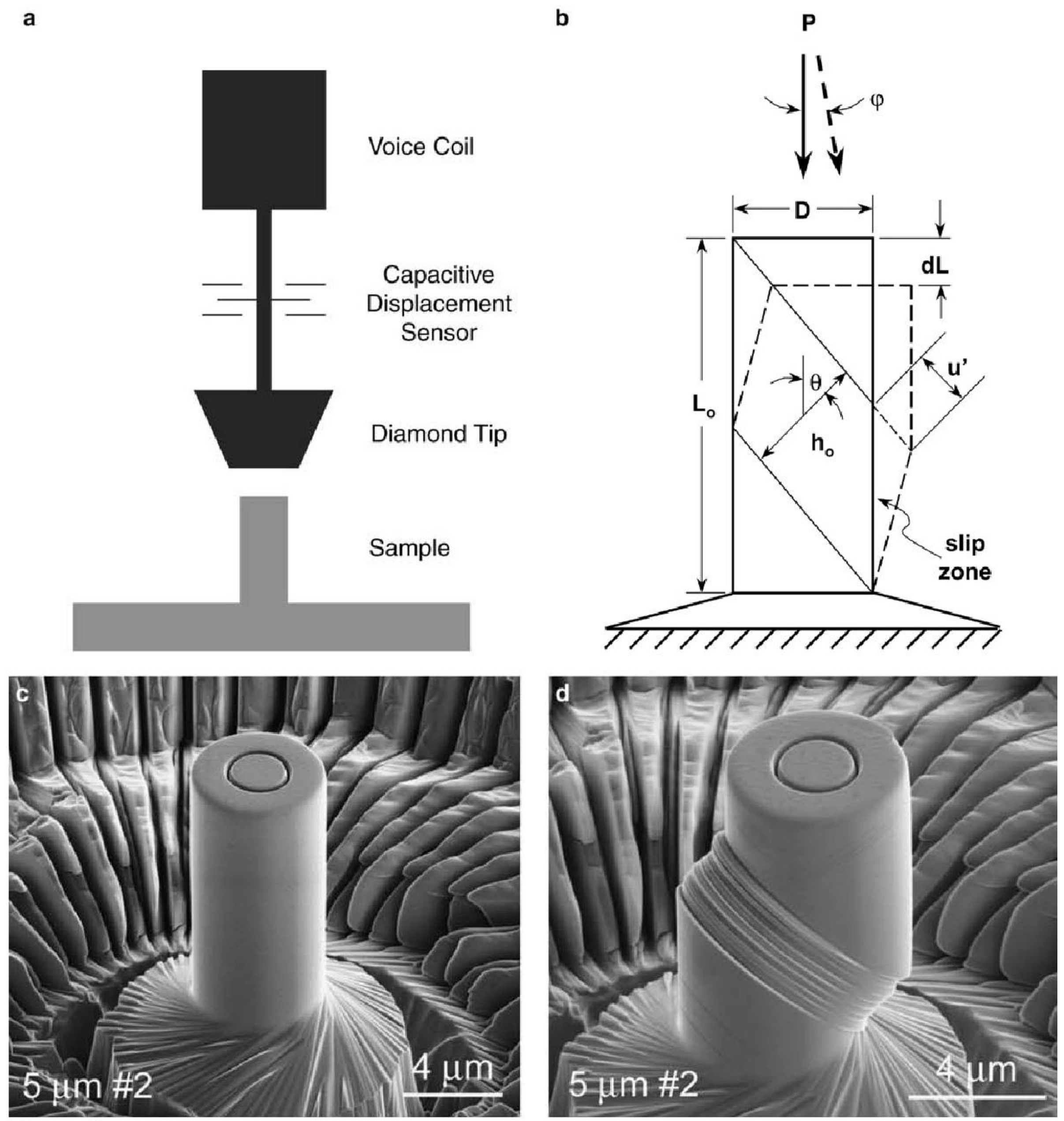
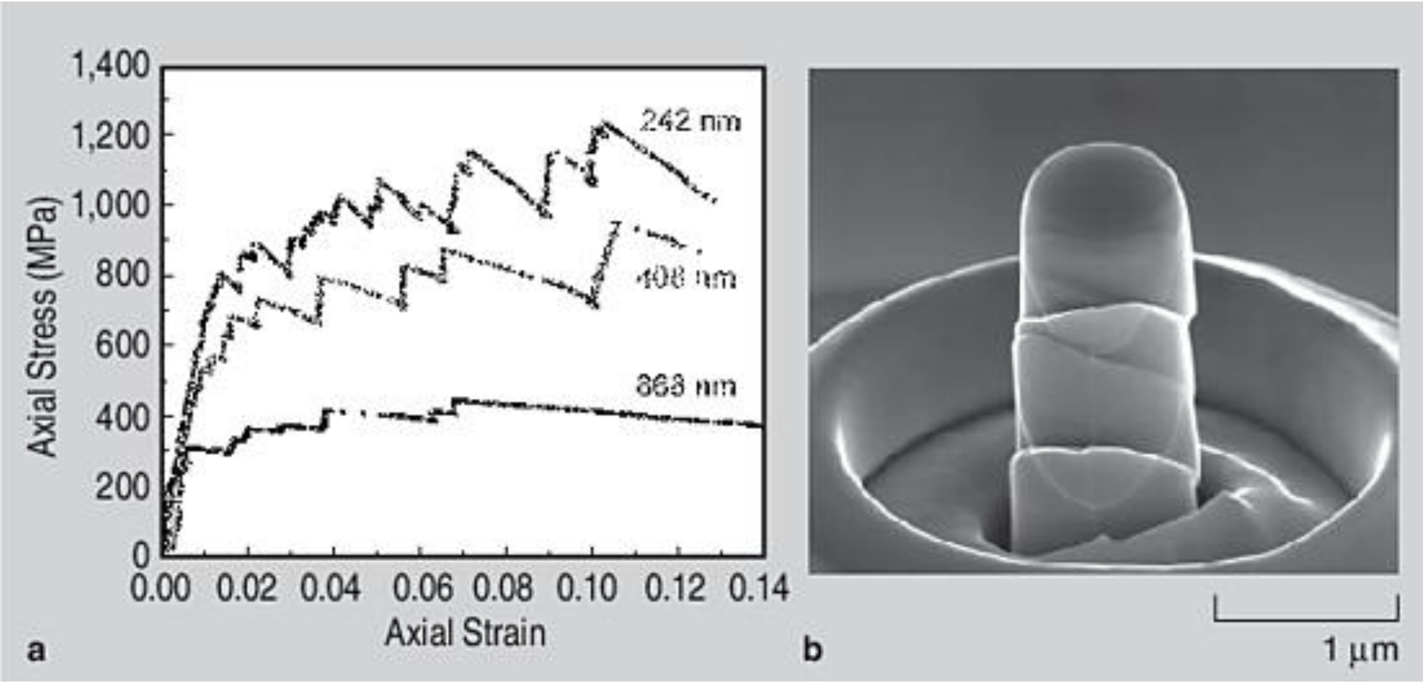
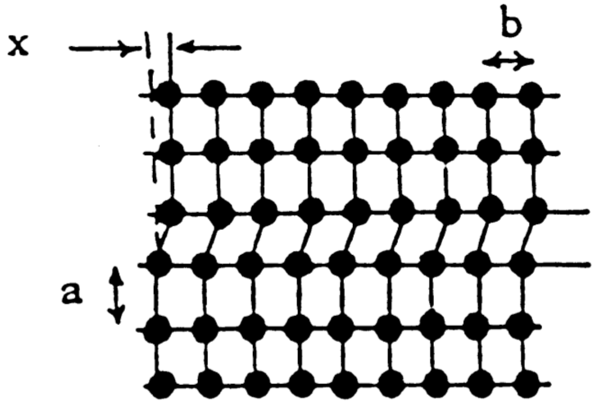


Figure 1

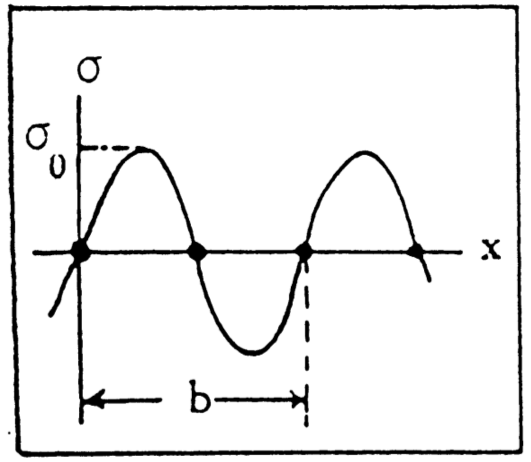


Serrated plasticity relates to the nucleation of dislocation proceeded by the gliding (at lower energy) on the slip plane (activated slip system) associated with the highest Schmid factor for this crystal system and orientation.

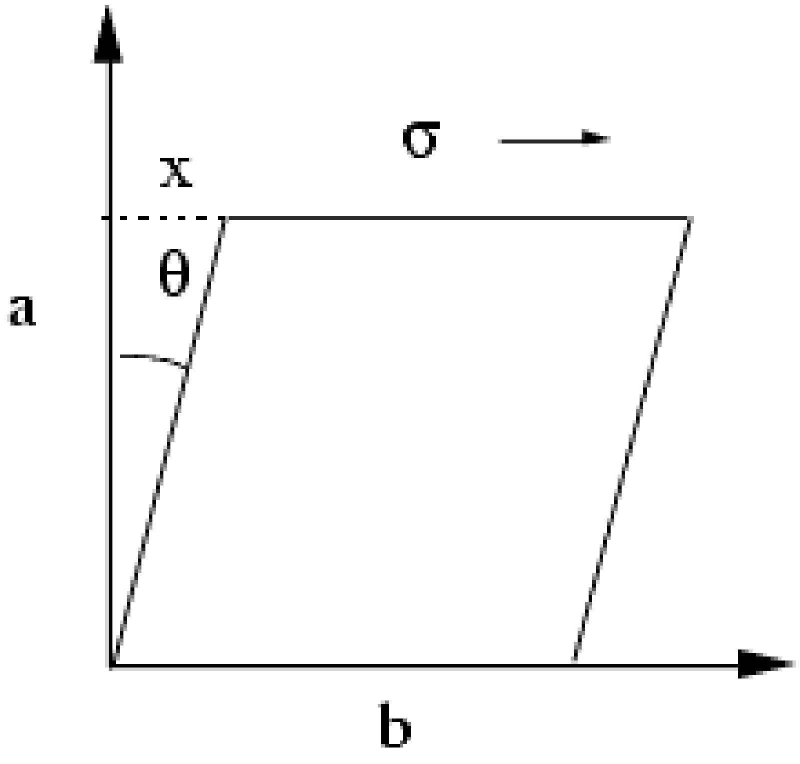
Shear-stress on a crystalline plane



a)



b)



$$\sigma = \sigma_0 \sin\left(\frac{2\pi x}{b}\right) \approx \sigma_0 \frac{2\pi x}{b}$$

$$\sigma = \mu\theta = \mu\frac{x}{a} \approx \sigma_0 \frac{2\pi x}{b} \qquad \sigma_0 = \frac{\mu b}{2\pi a}$$

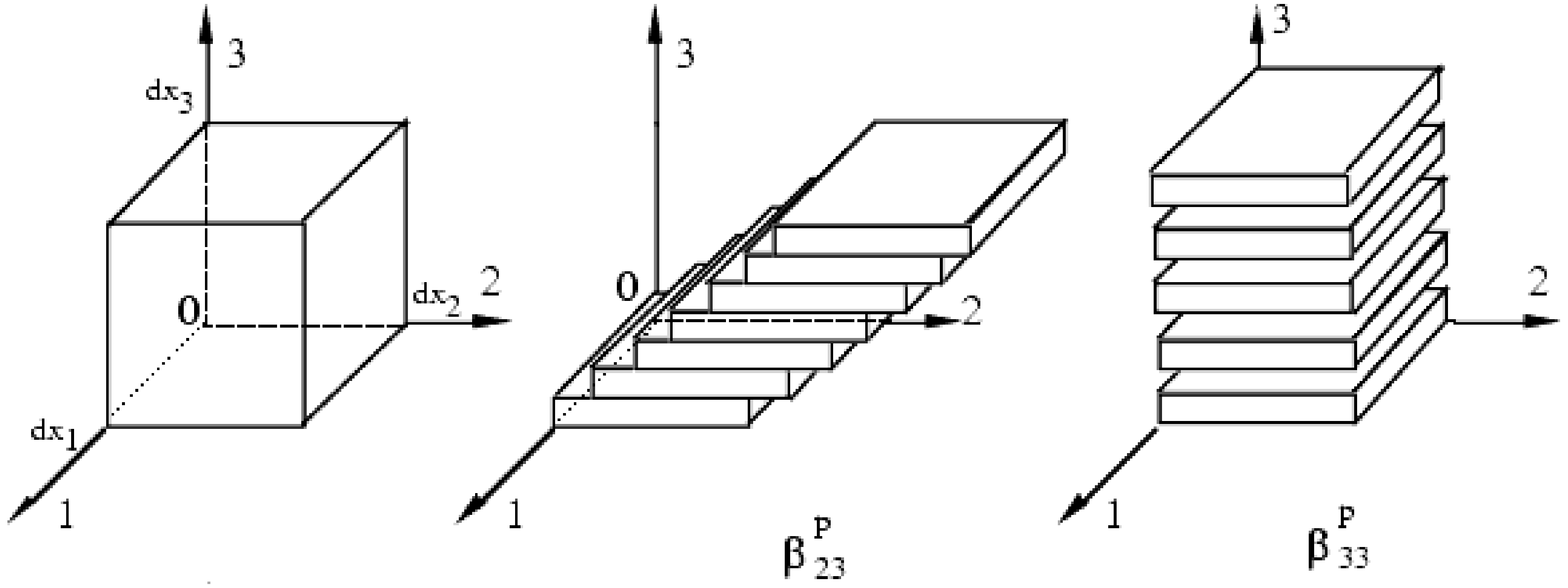
$$\sigma = \frac{\mu b}{2\pi a}\sin\left(\frac{2\pi x}{b}\right)$$ Maximum at $x = {}^{b}/_{4}, a > b$

For copper

$$\sigma_{cr} \sim \frac{\mu}{10} = 4.5\ GPa\ but\ \sigma_y \sim 30\ MPa$$

Frenkel Limit

Plastic distortion



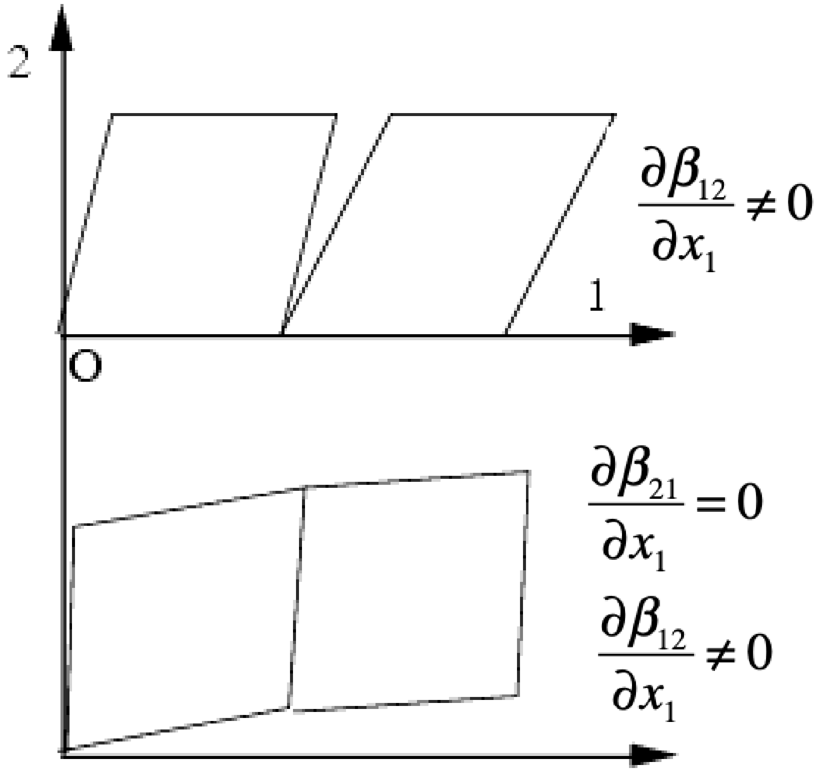
$$du_i^p = \int_0^{dx_j} \left(\frac{\partial u_i^p}{\partial x_j} \right) \partial x_j = \left(\frac{\partial u_i^p}{\partial x_j} \right) dx_j \qquad du_i^p = \beta_{i1}^p dx_1 + \beta_{i2}^p dx_2 + \beta_{i3}^p dx_3 \qquad d\vec{u}^{\,p} = \overline{\overline{\beta}}^{\,p} d\vec{x}$$

$$d\vec{u} = d\vec{u}^{\,p} + d\vec{u}^{\,e}$$

Incompatible distortion: cracks

Compatible distortion= no cracks

$$\frac{\partial \beta_{21}}{\partial x_1} = \frac{\partial \beta_{31}}{\partial x_1} = 0$$



Plastic distortion

$d\vec{u}$ is an exact total differential thus:

$$d\vec{u} = \frac{\partial \vec{u}}{\partial x_j} dx_j = \vec{\beta}_j dx_j$$

Cauchy identity $\frac{\partial \vec{\beta}_j}{\partial x_k} = \frac{\partial \vec{\beta}_k}{\partial x_j}$ if $j \neq k$

↓

Physical meaning, the creation of defects during plastic deformation creates residual internal stresses

$$\begin{bmatrix} \frac{\partial \vec{\beta}_3}{\partial x_2} - \frac{\partial \vec{\beta}_2}{\partial x_3} \\ \frac{\partial \vec{\beta}_1}{\partial x_3} - \frac{\partial \vec{\beta}_3}{\partial x_1} \\ \frac{\partial \vec{\beta}_2}{\partial x_1} - \frac{\partial \vec{\beta}_1}{\partial x_2} \end{bmatrix} = 0 = \overrightarrow{rot}\,\overline{\overline{\beta}}$$

No cracks!

$$\overrightarrow{rot}\,\overline{\overline{\beta^e}} = -\overrightarrow{rot}\,\overline{\overline{\beta^p}} = \overline{\overline{\alpha}}$$

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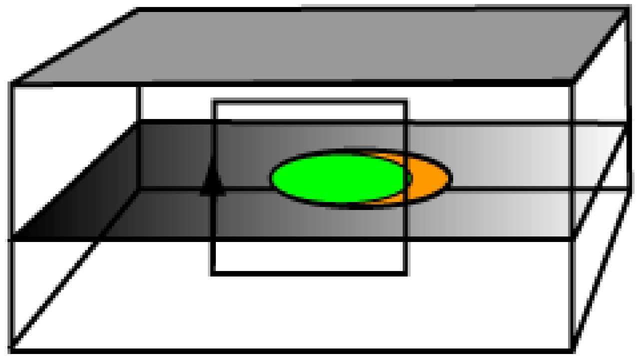
Compatibility

$$\overline{\overline{\alpha}} = \overrightarrow{rot}\,\overline{\overline{\beta^e}} = -\overrightarrow{rot}\,\overline{\overline{\beta^p}}$$

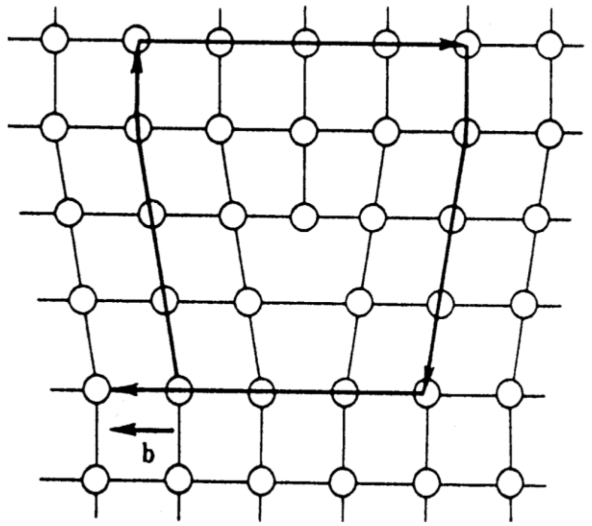
$$\iint \overline{\overline{\alpha}}\, d\vec{S} = \iint \overrightarrow{rot}\,\overline{\overline{\beta^e}}\, d\vec{S} = \oint \overline{\overline{\beta^e}}\, d\vec{x} = \oint \vec{\beta}^e_j dx_j = \oint d\overrightarrow{u^e}$$

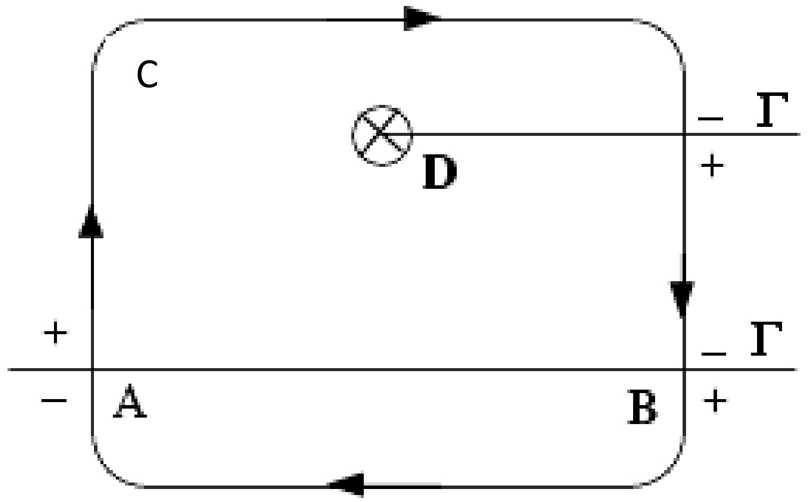


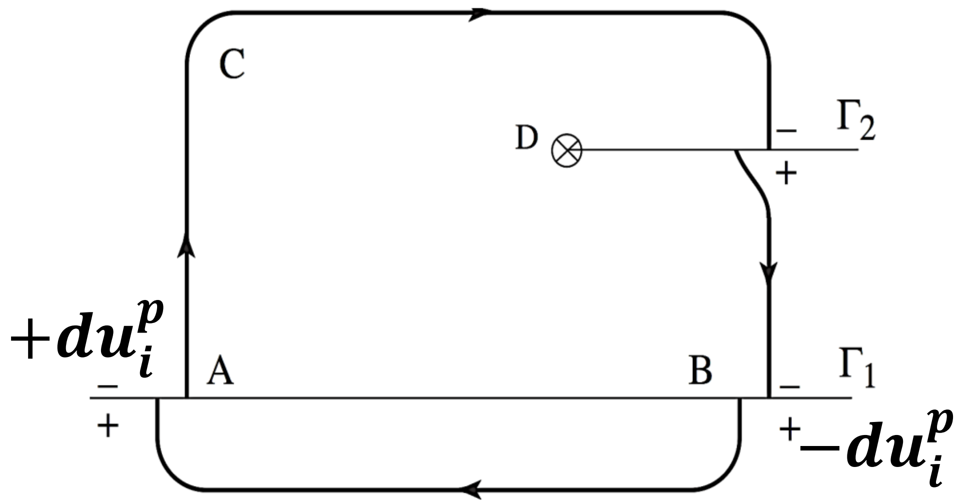
Burgers vector

$$\boxed{\vec{b} = \oint d\overrightarrow{u^e} = -\oint d\overrightarrow{u^p} = \iint \overline{\overline{\alpha}}\, d\vec{S}}$$

$$\alpha_{ij} = \frac{\Delta b_i}{\Delta S_j}$$







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Analogy with electromagnetism

$$\vec{b} = \oint_C \bar{\bar{\beta}}^e \, d\vec{x} \qquad i = \oint_C \vec{H} \, d\vec{l}$$

$$\vec{b} \leftrightarrow i \qquad \bar{\bar{\beta}}^e \leftrightarrow \vec{H}$$

$$\bar{\bar{\alpha}} = rot \;\; \beta^e \qquad \vec{j} = rot \;\; \vec{H}$$

$$div \;\; \bar{\bar{\alpha}} = 0 \qquad div \; \vec{j} = 0$$

Dislocations induce internal distortion fields like current flowing in wire induces magnetic fields

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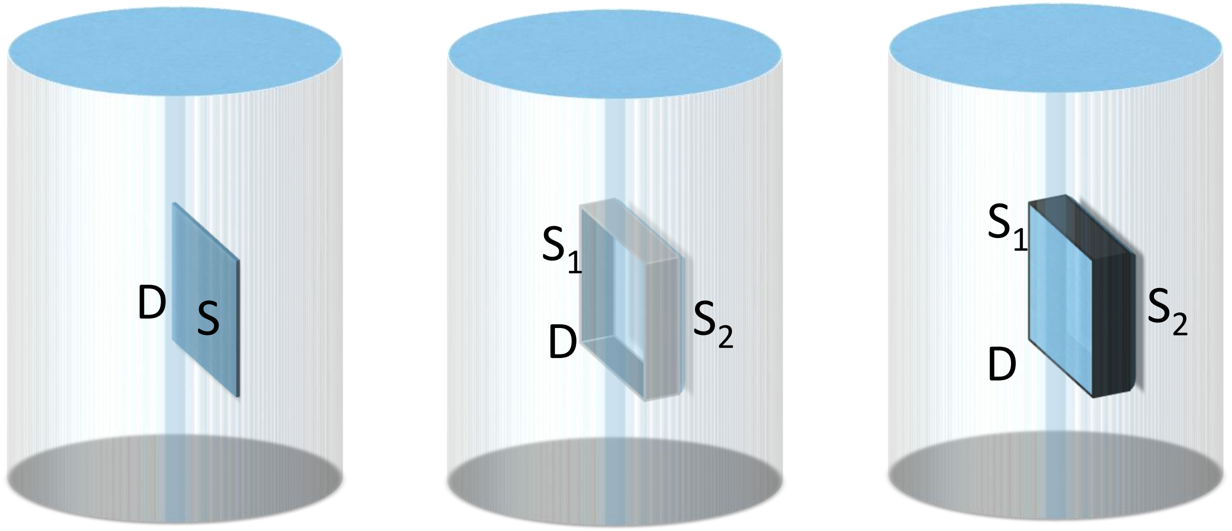
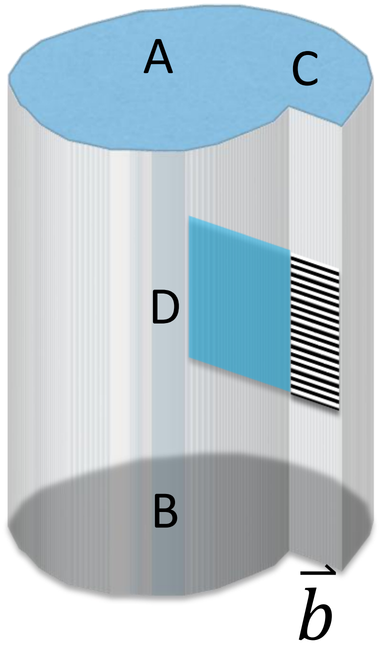
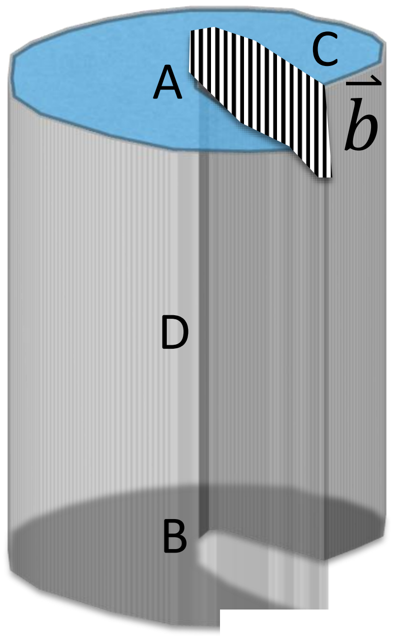
Continuum: Volterra dislocations (1860-1940)

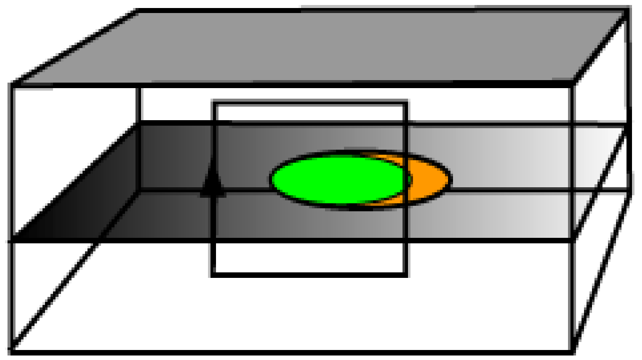




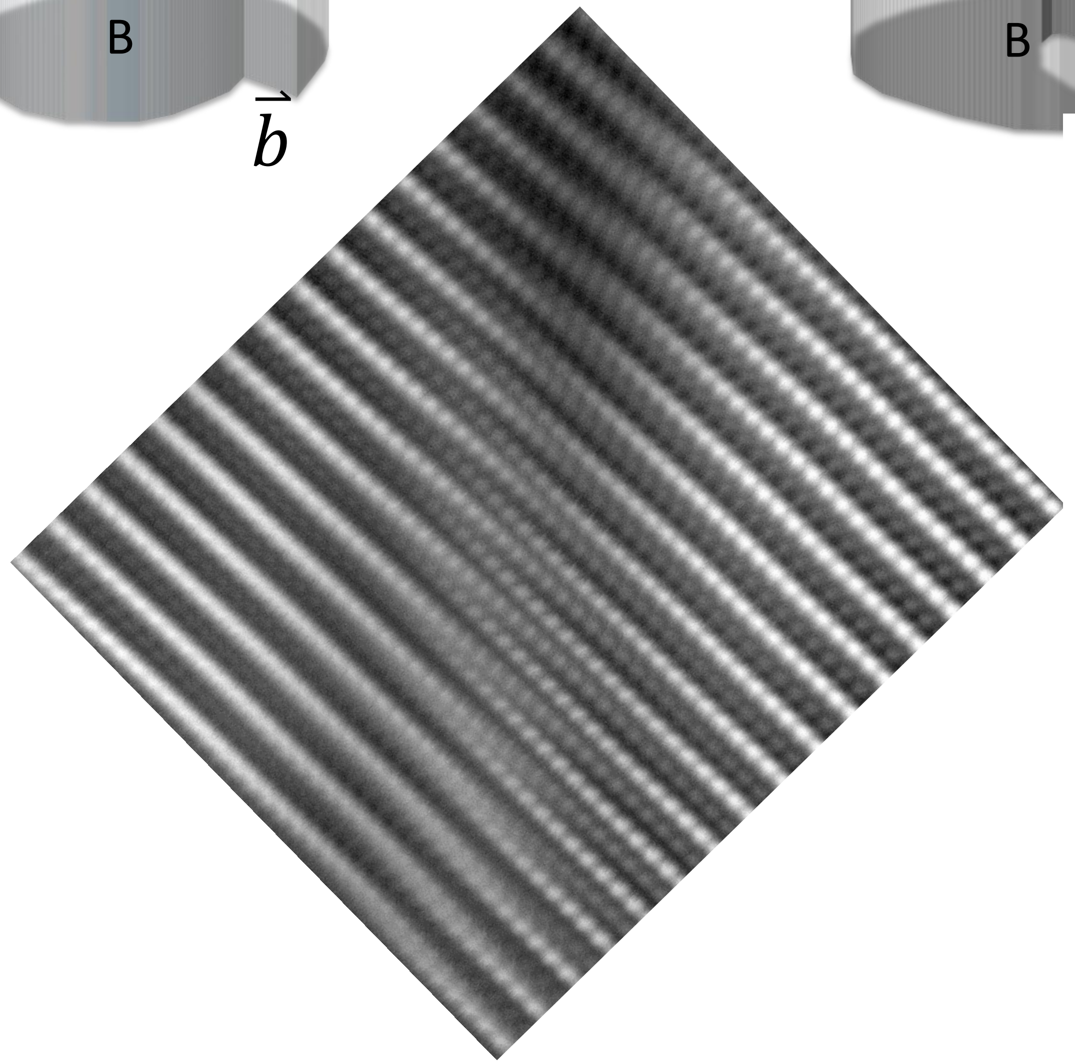
Edge dislocation

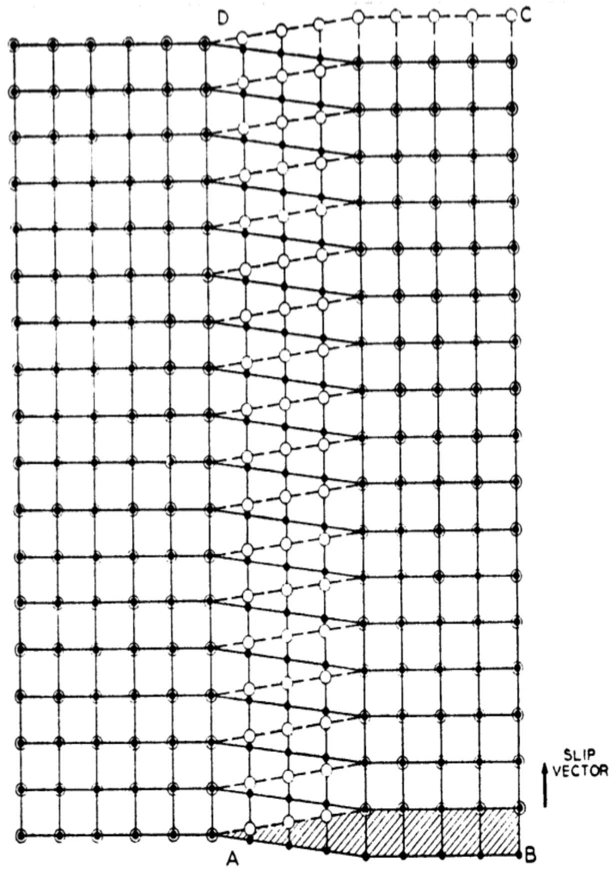


Screw dislocation



cut, shear, paste





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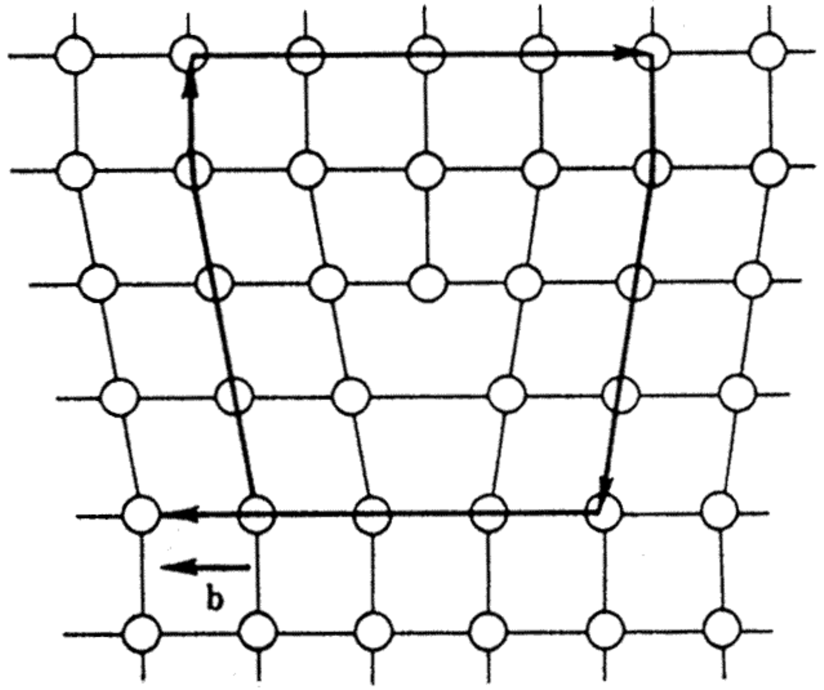
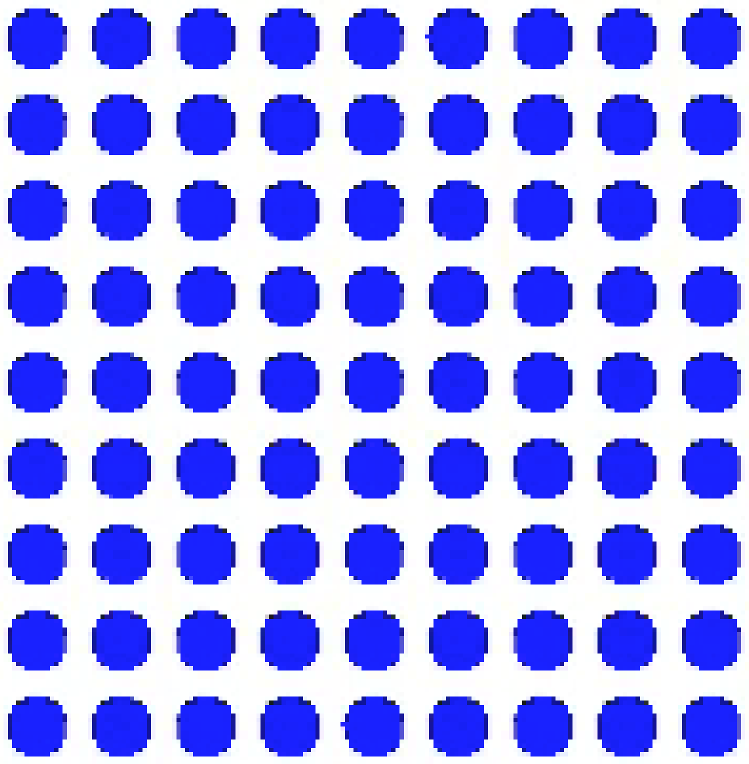
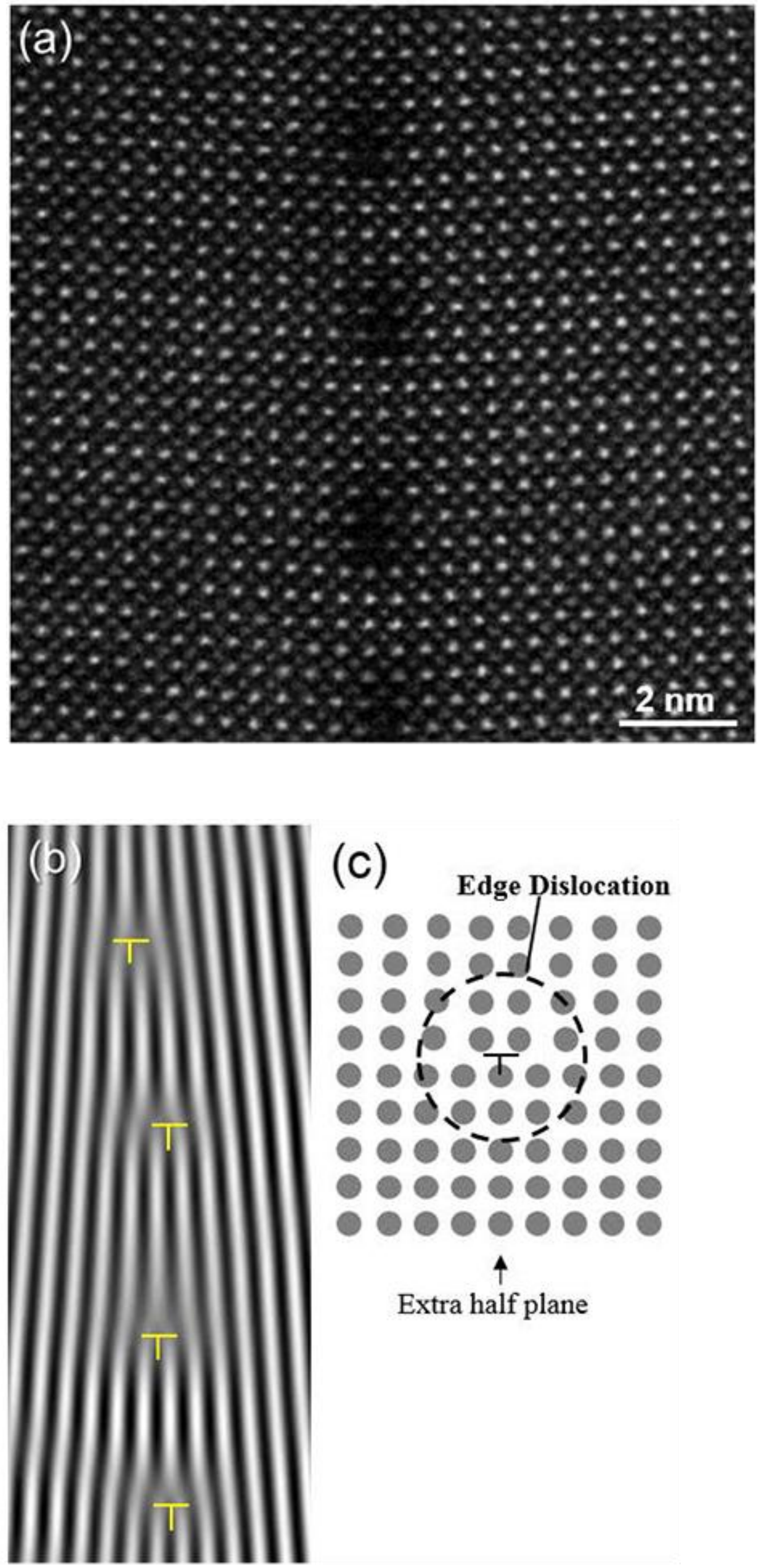
Transposition to the crystalline lattice: Burgers vectors







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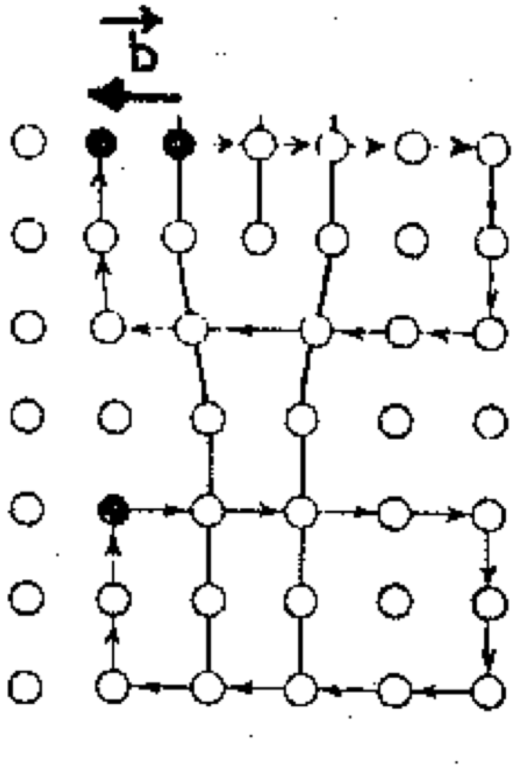
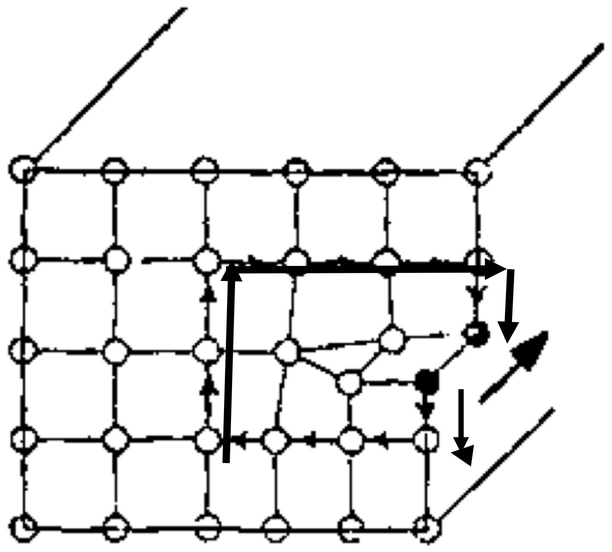
Slip plane



$\vec{b}$ and $\vec{\xi}$ are perpendicular

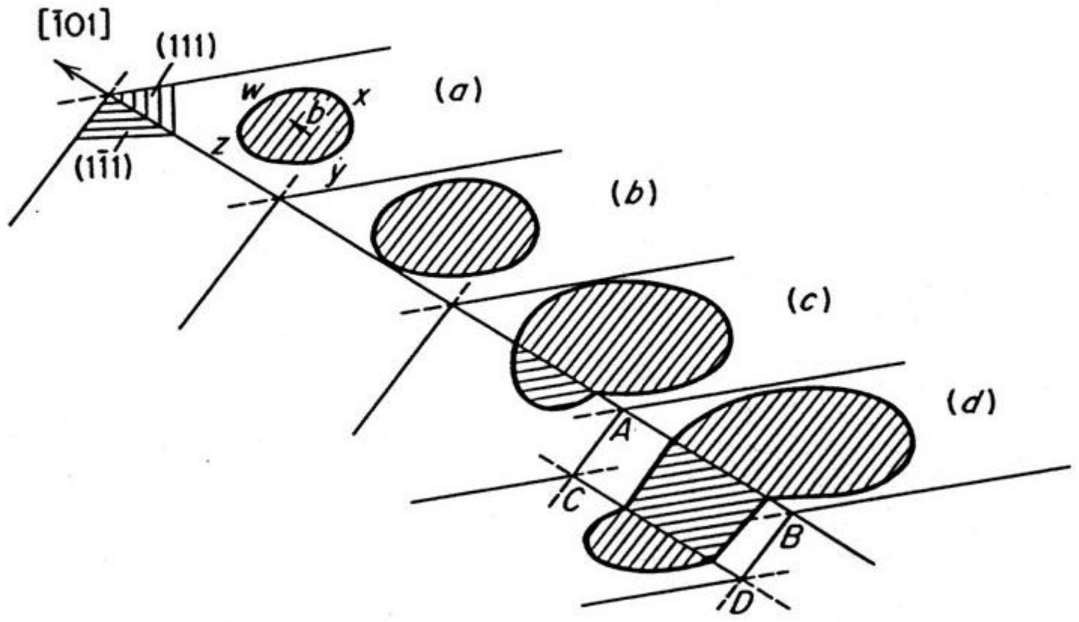
Edge



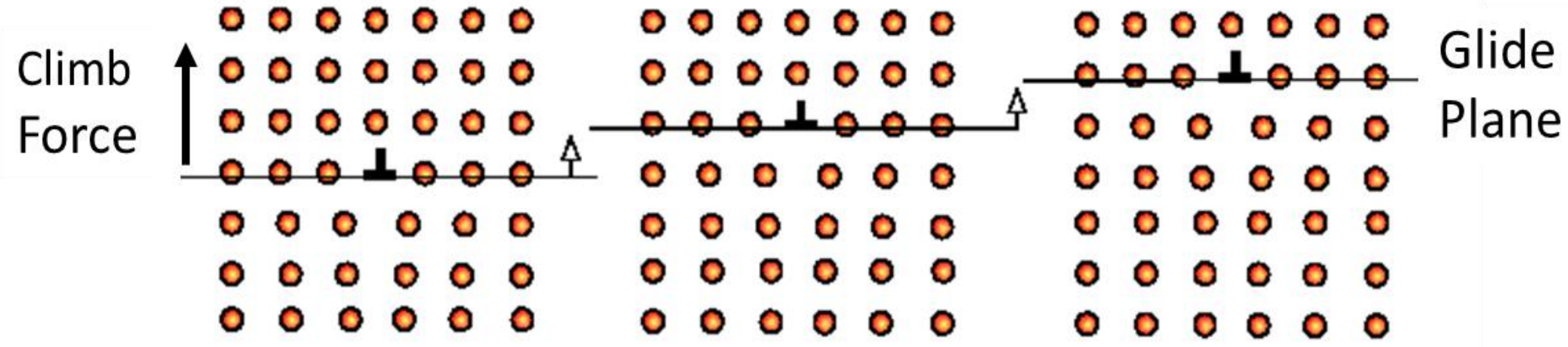
$\vec{b}$ and $\vec{\xi}$ are parallel

Screw

Cross slip



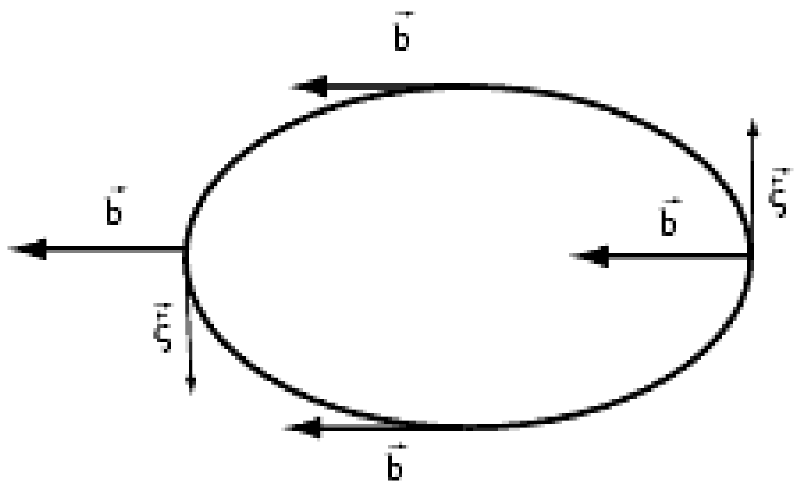
Climb



Properties of the Burgers vector

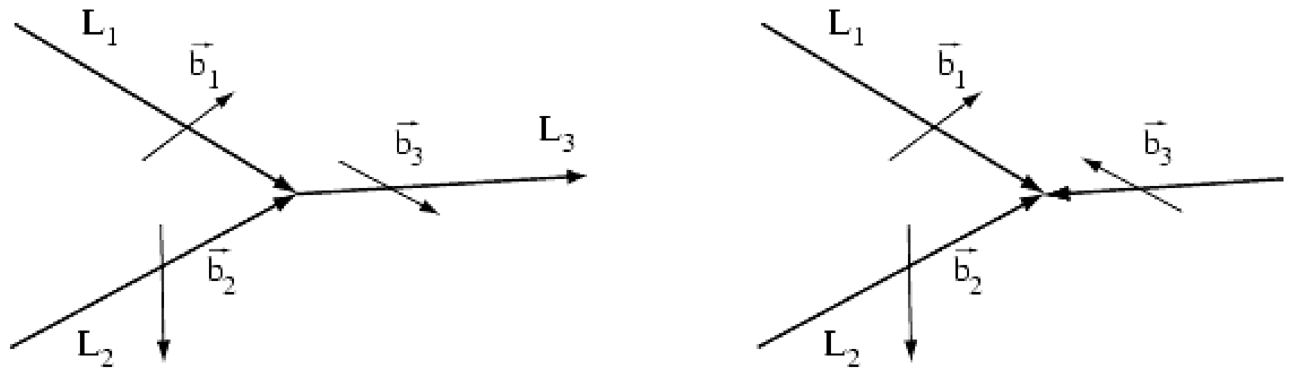
Equivalent of the Kirchhoff laws

Conservation (mesh rule)



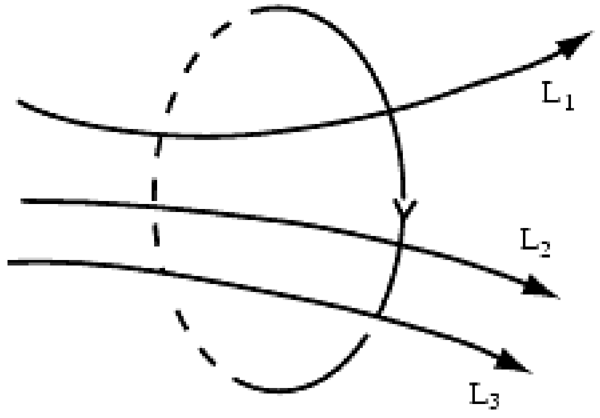
$\vec{b}$ is constant

Sum at a node



$$\vec{b_3} = \vec{b_1} + \vec{b_2}$$

additivity



$$\vec{b} = \vec{b_1} + \vec{b_2} + \vec{b_3}$$

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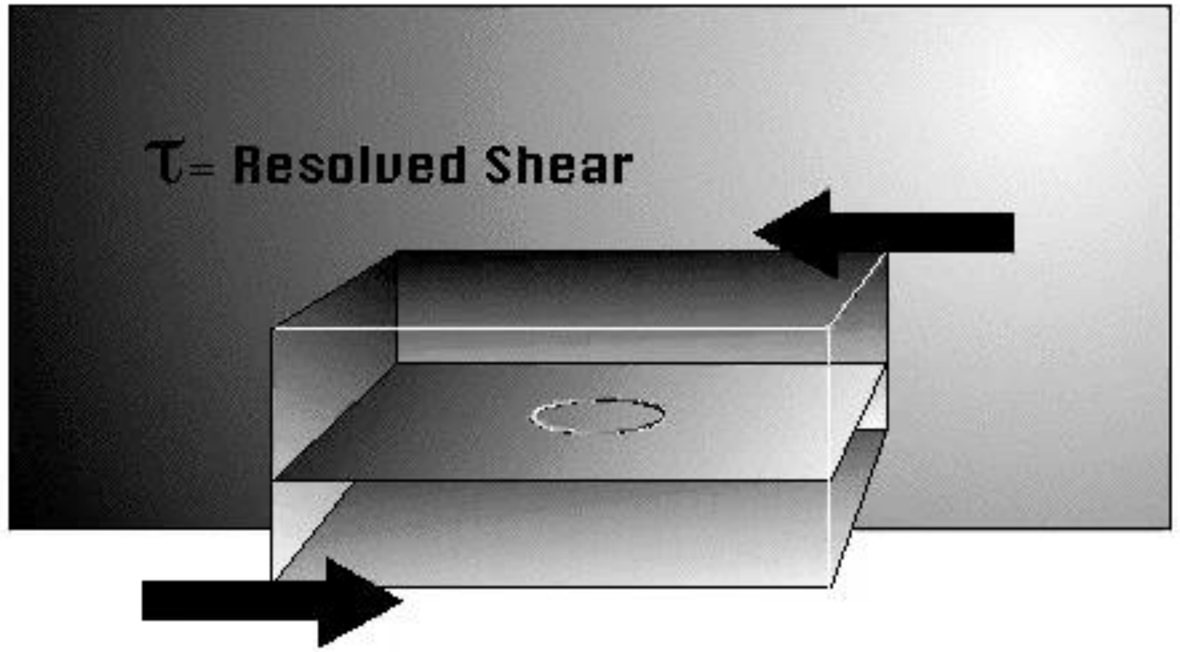
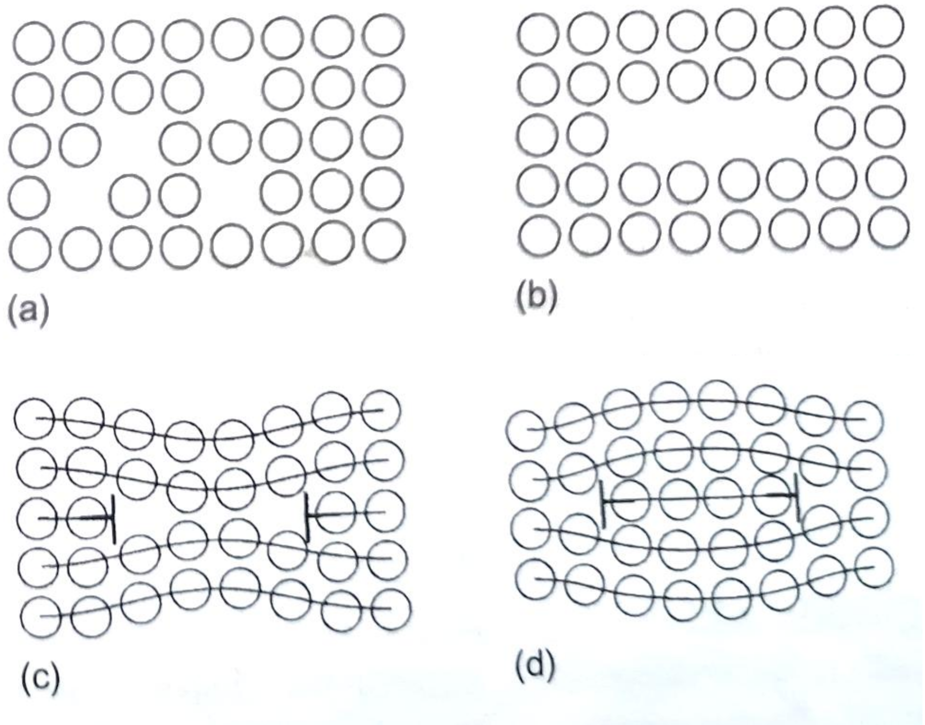
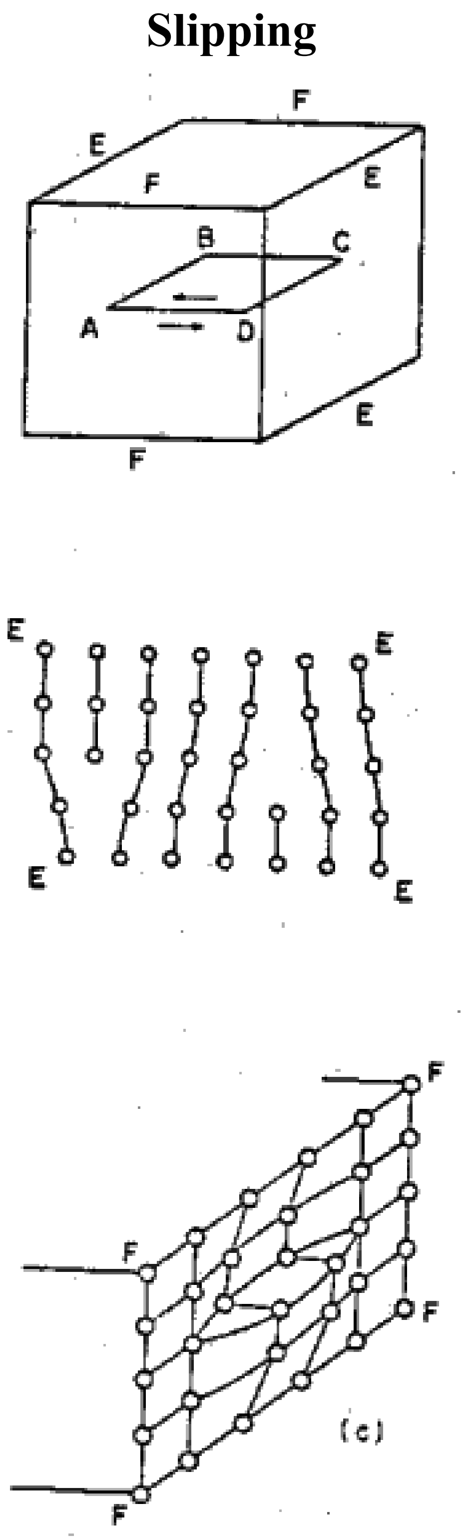
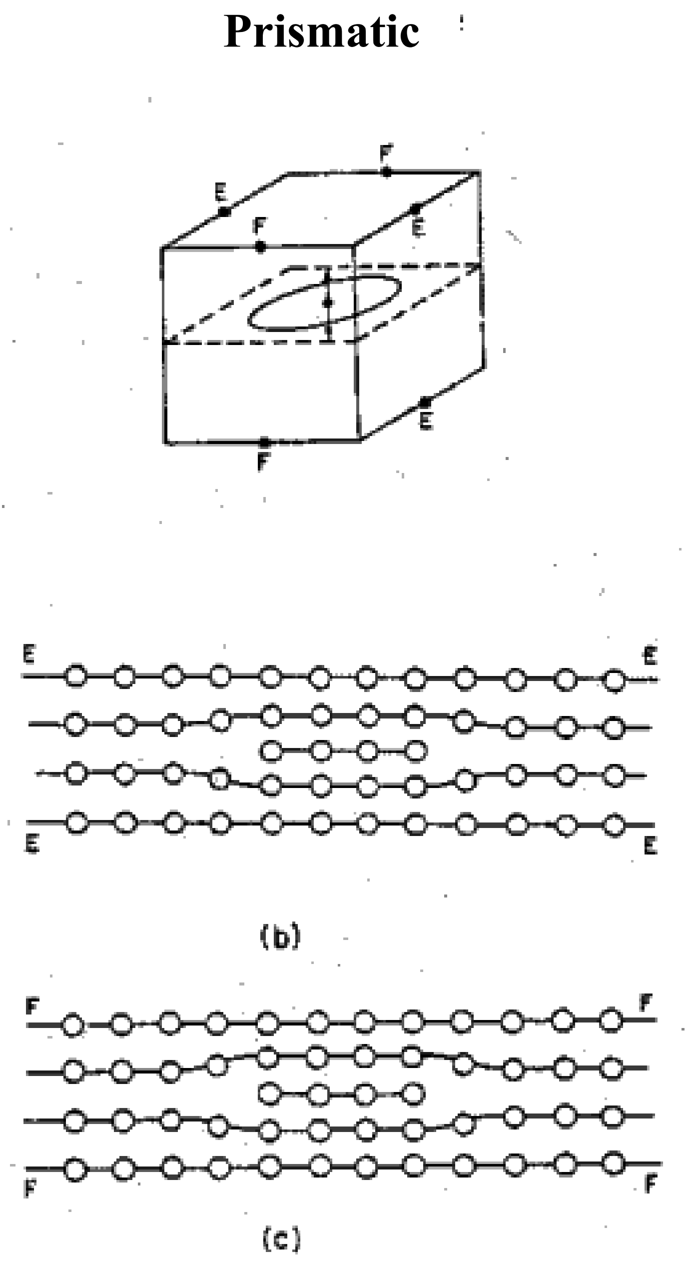
Dislocation loops



Formation of dislocation loops
in irradiation







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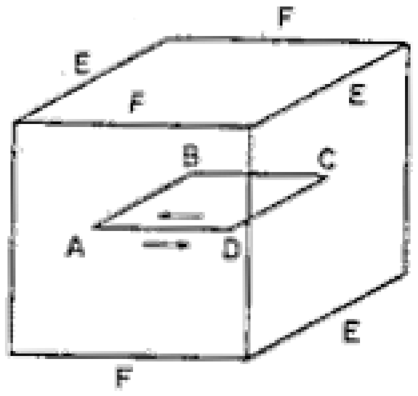
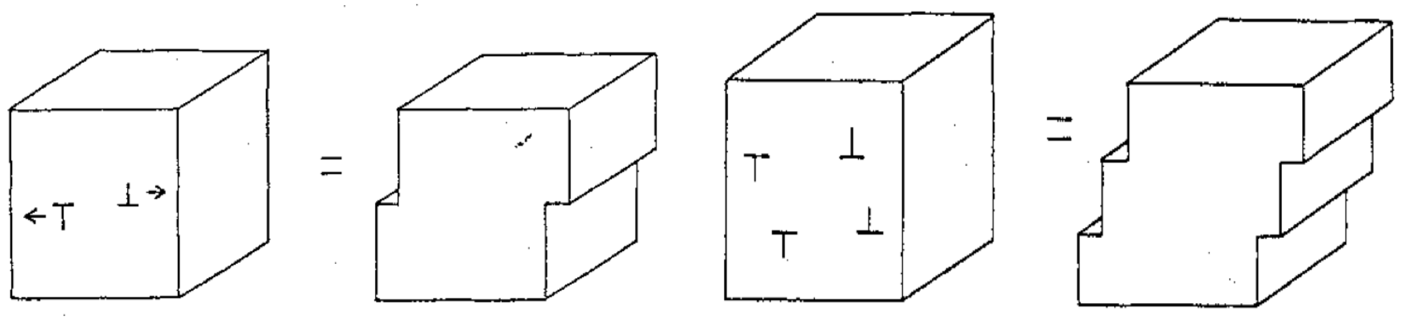
Dislocation loops

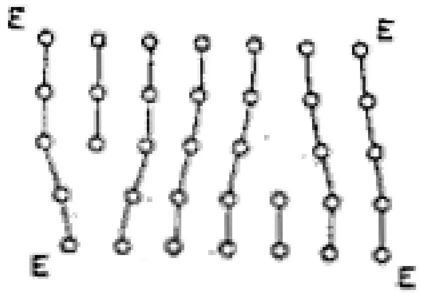
Slip

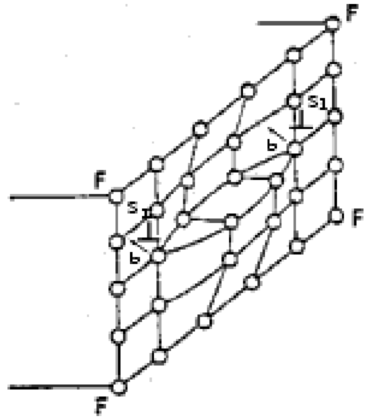


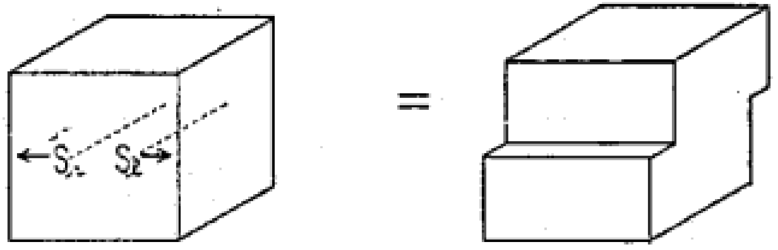
Edge components

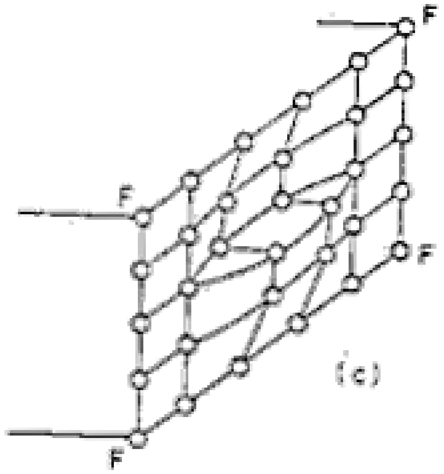


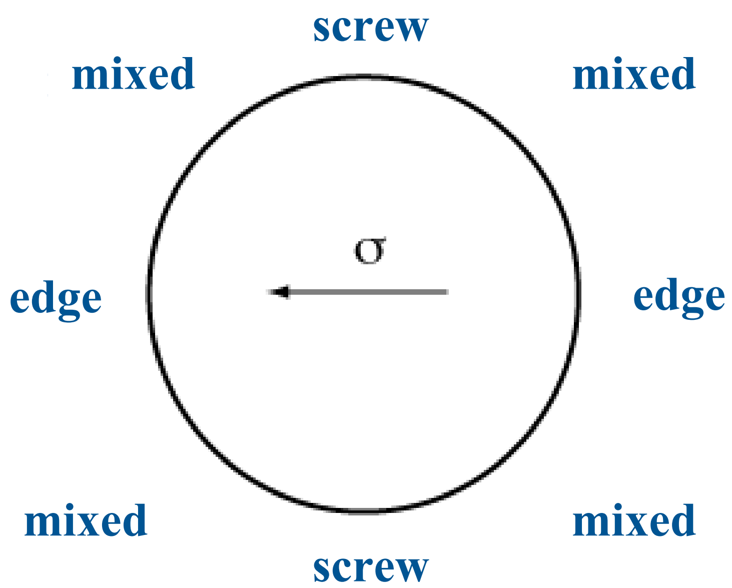
Screw components

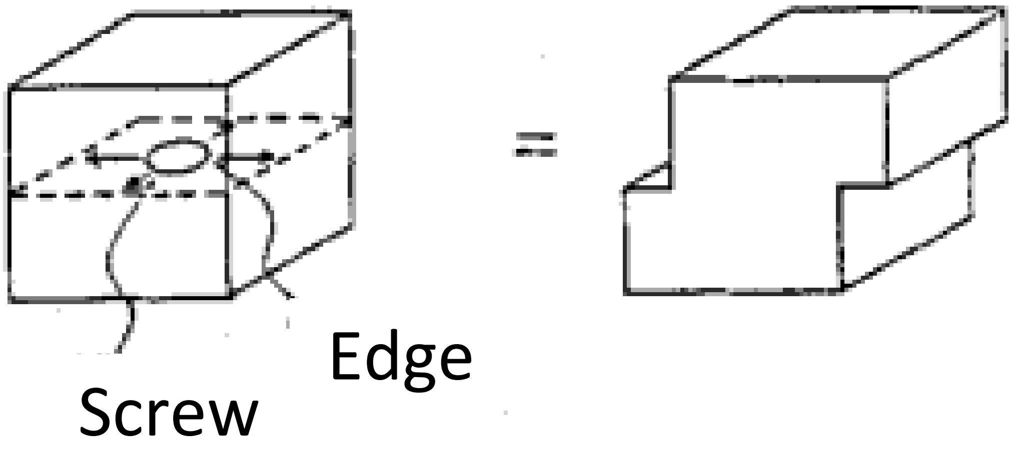












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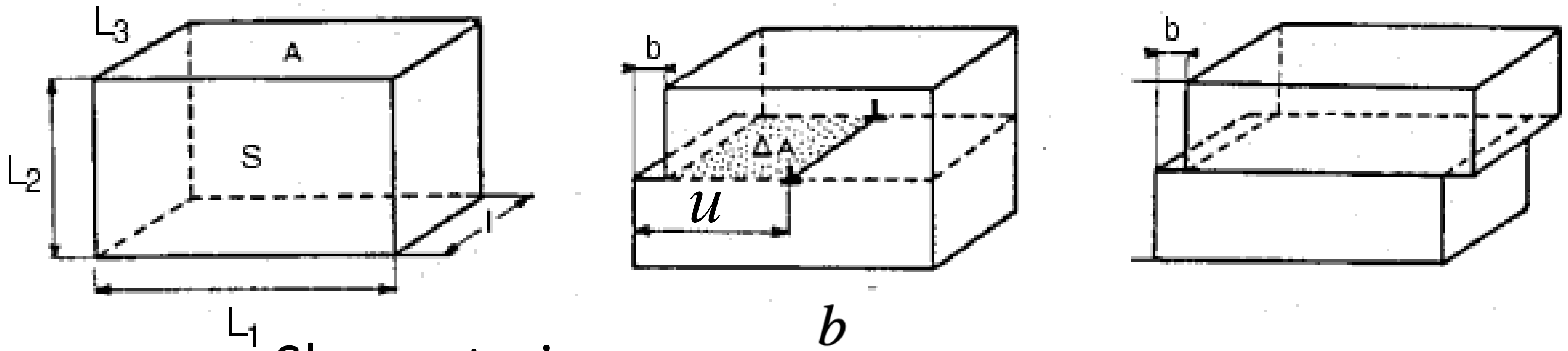
Orowan equation

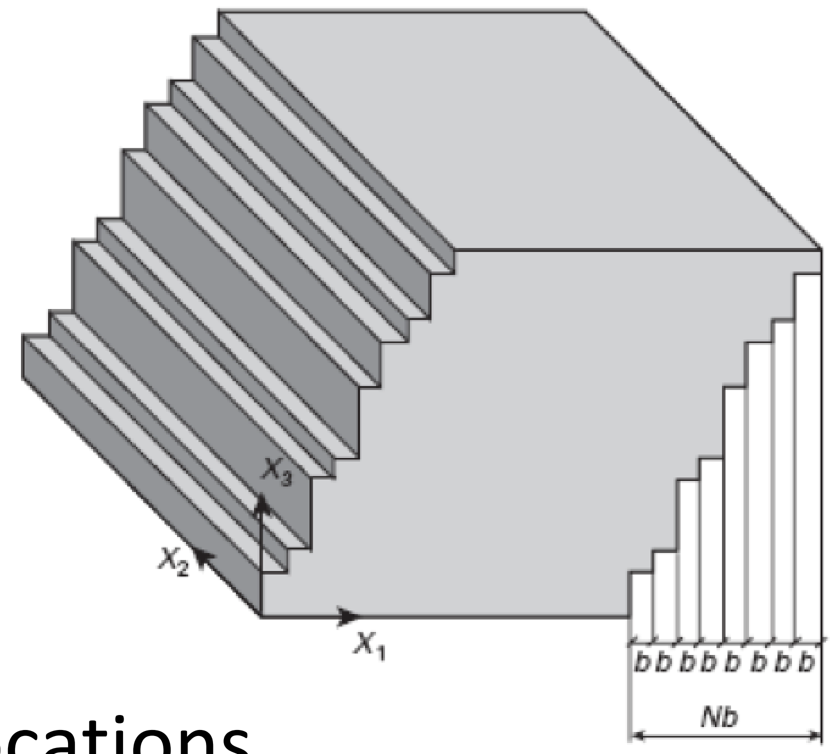


Shear-strain: $\varepsilon = \frac{b}{L_2}$

Partial displacement

$$\varepsilon = \frac{b}{L_2}\frac{u}{L_1} = \frac{b}{S}u$$

$$\varepsilon = \frac{b}{L_2}\frac{u}{L_1}\frac{L_3}{L_3} = \frac{b}{V}\Delta A$$



Many dislocations: density of dislocations

$$\varepsilon = \frac{N}{V}b\cdot\Delta A \qquad \varepsilon = \frac{N}{S}b\cdot u \qquad \Lambda = \frac{N}{S} = \frac{N\cdot L_3}{V}$$

$$\dot{\varepsilon} = \Lambda b\dot{u} = \Lambda b v$$

Orowan's equation

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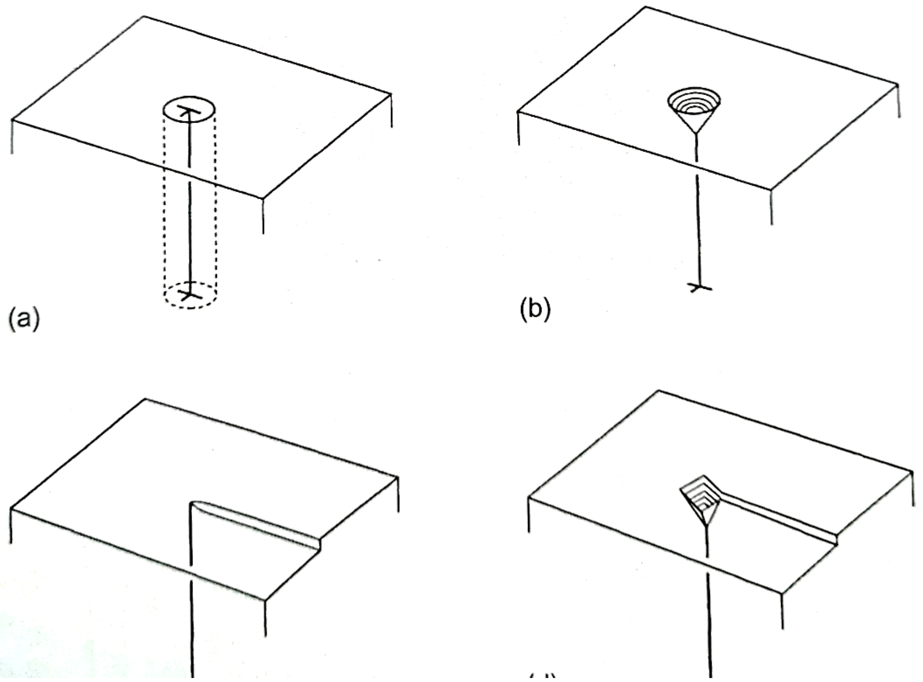
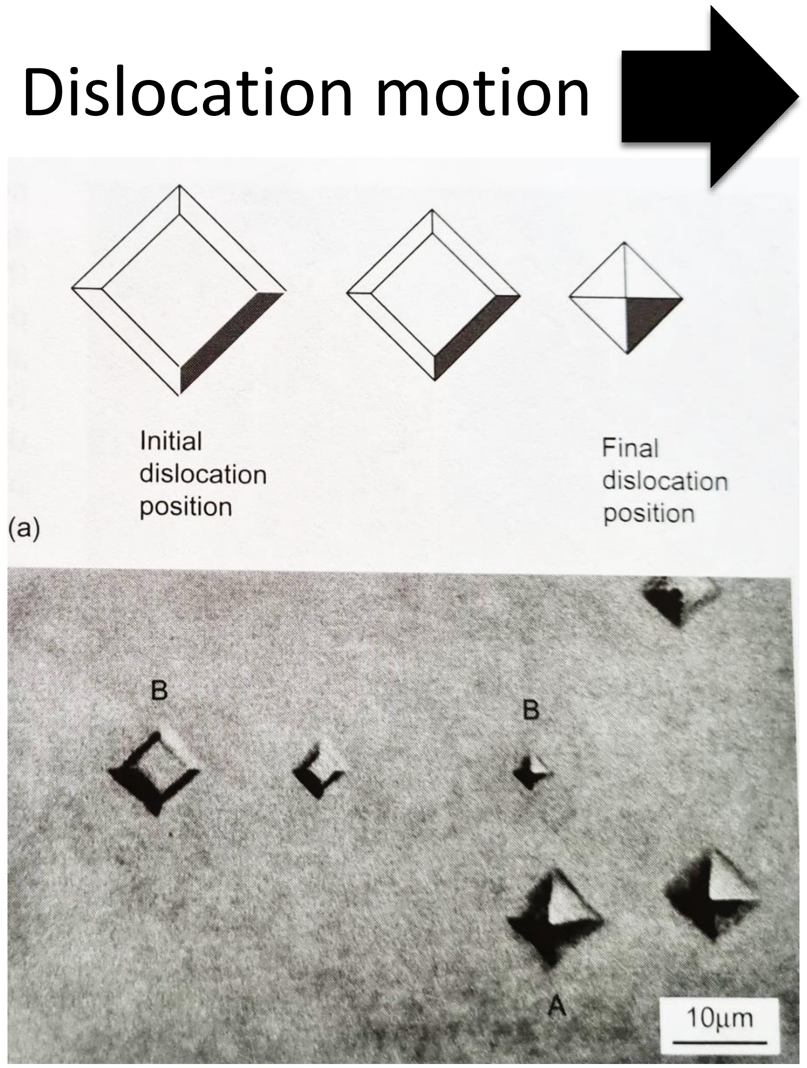
Etch pits

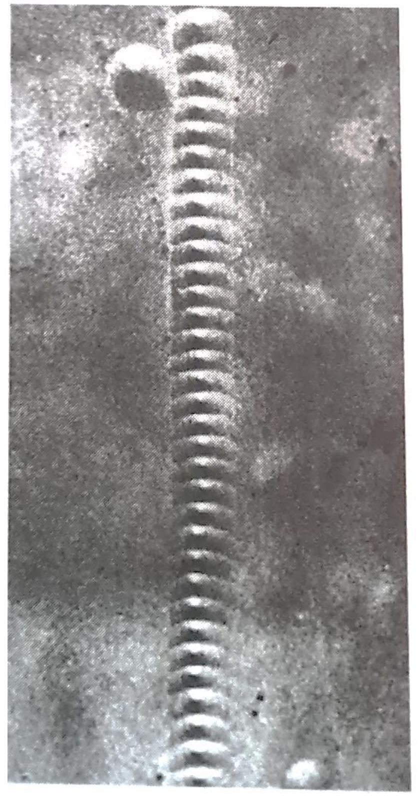
Edge

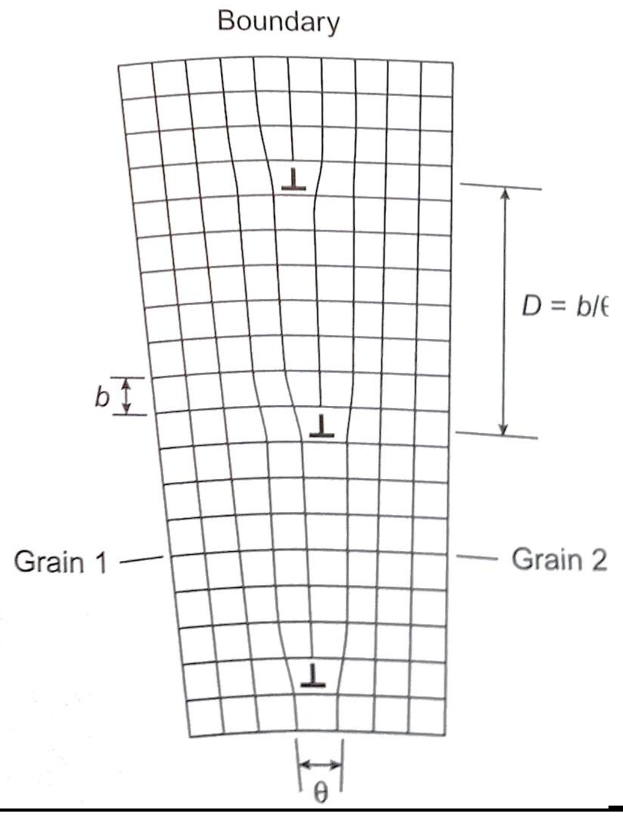
Screw



Dislocation motion

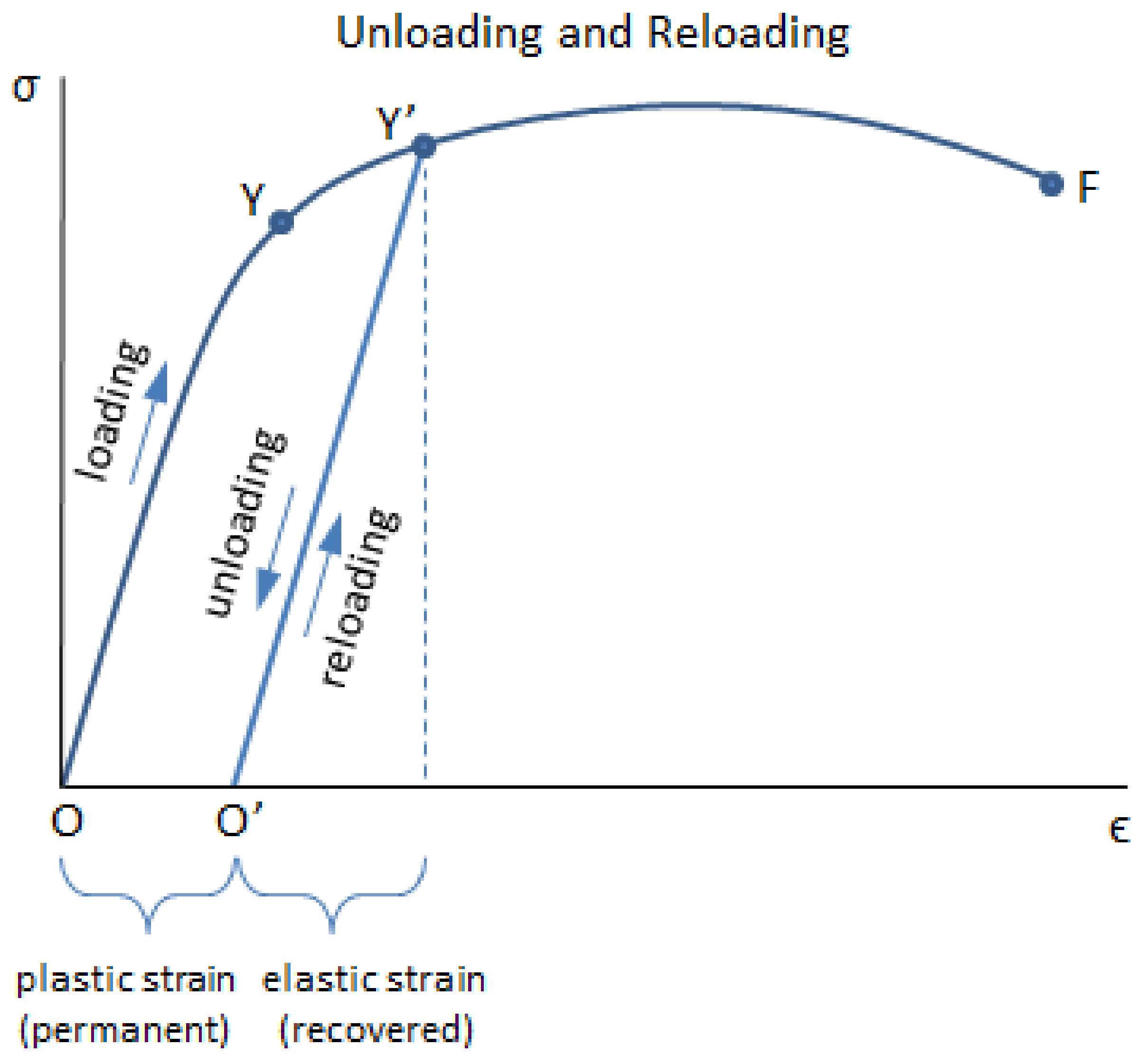




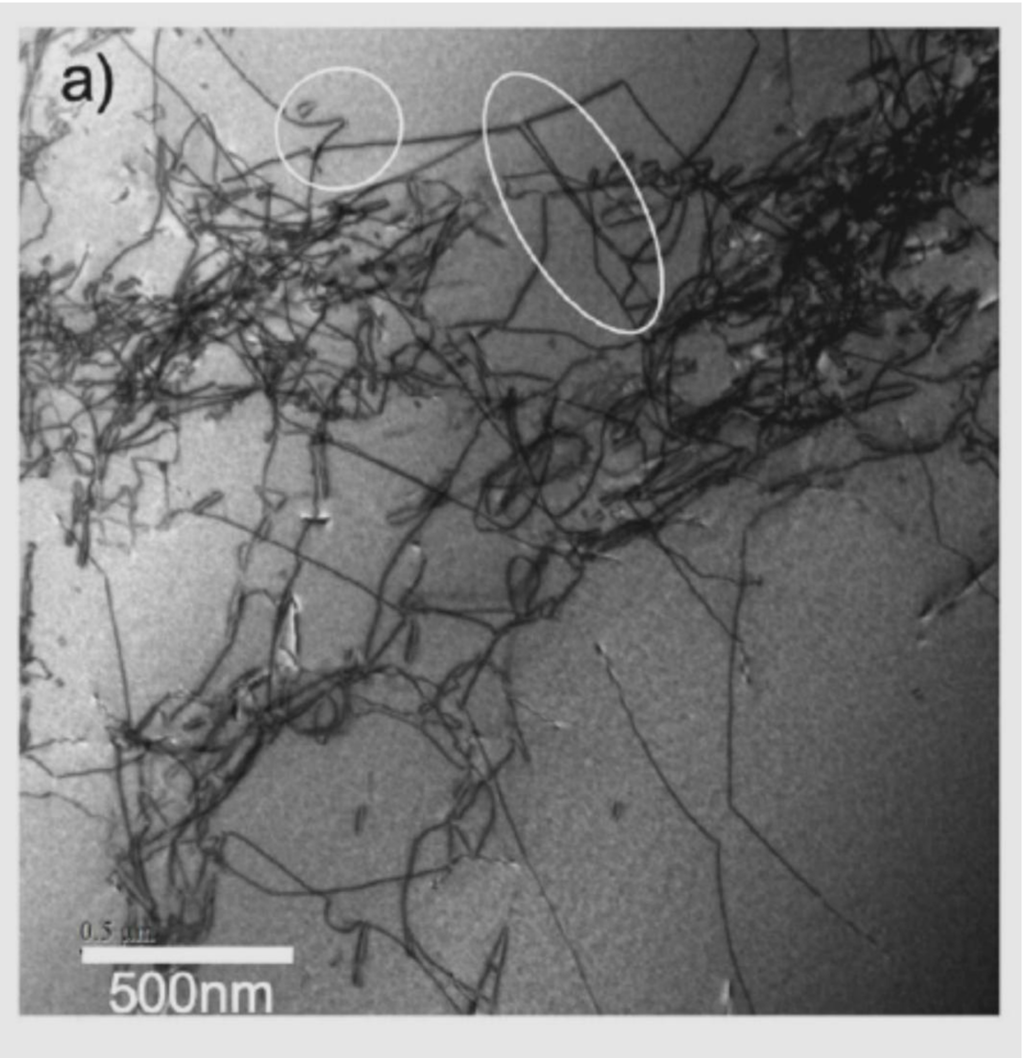


Etch pits along the grain boundaries having a dislocation network

Strain hardening: multiplying dislocations



Tensile curve:
increase of the elastic limit



Dislocations in 10% deformed iron

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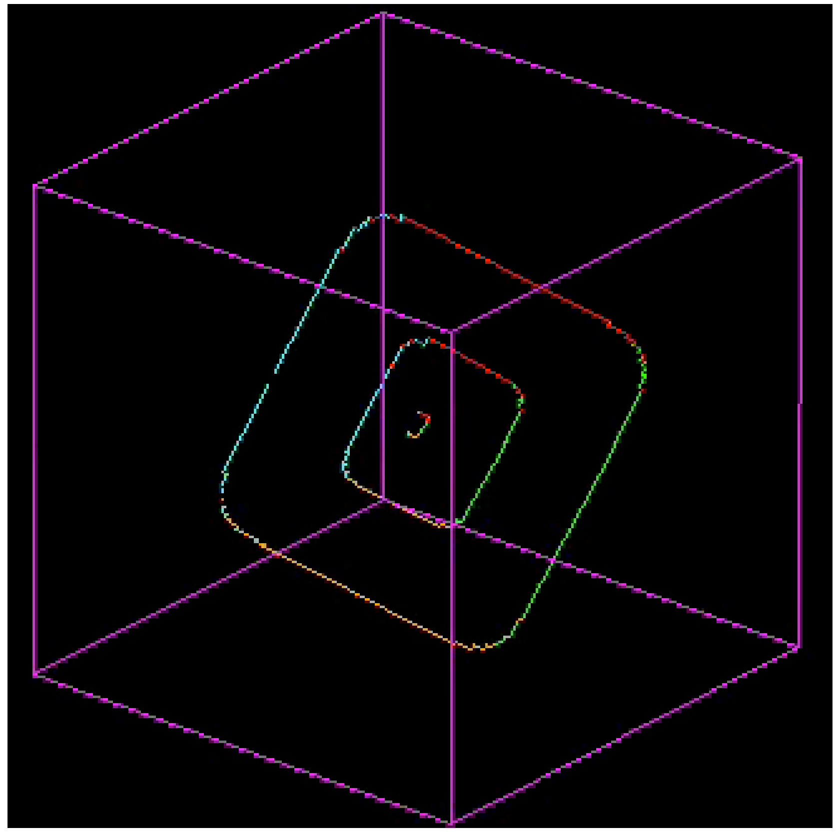
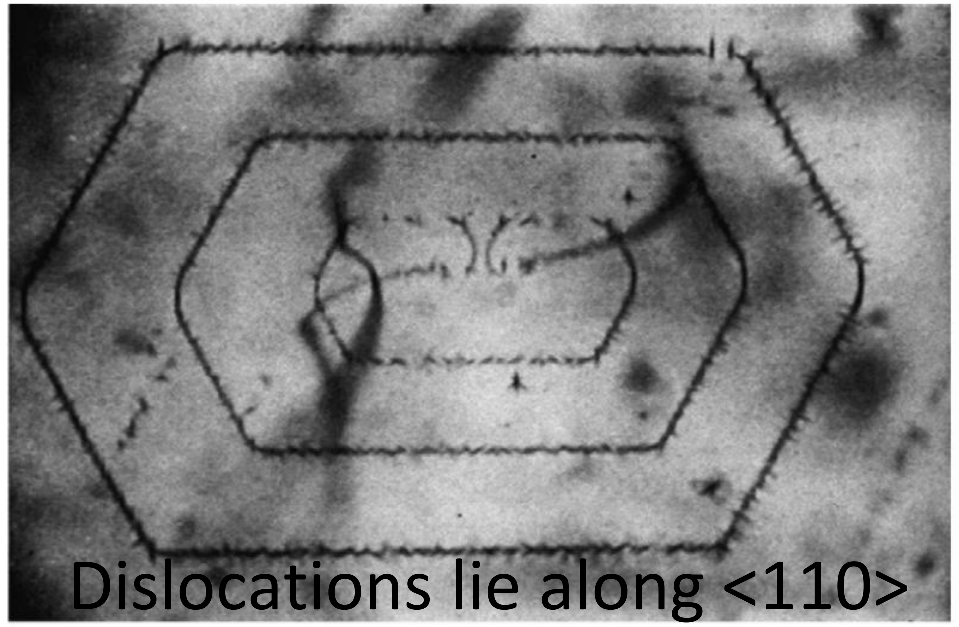
Multiplying dislocations: Frank-Read

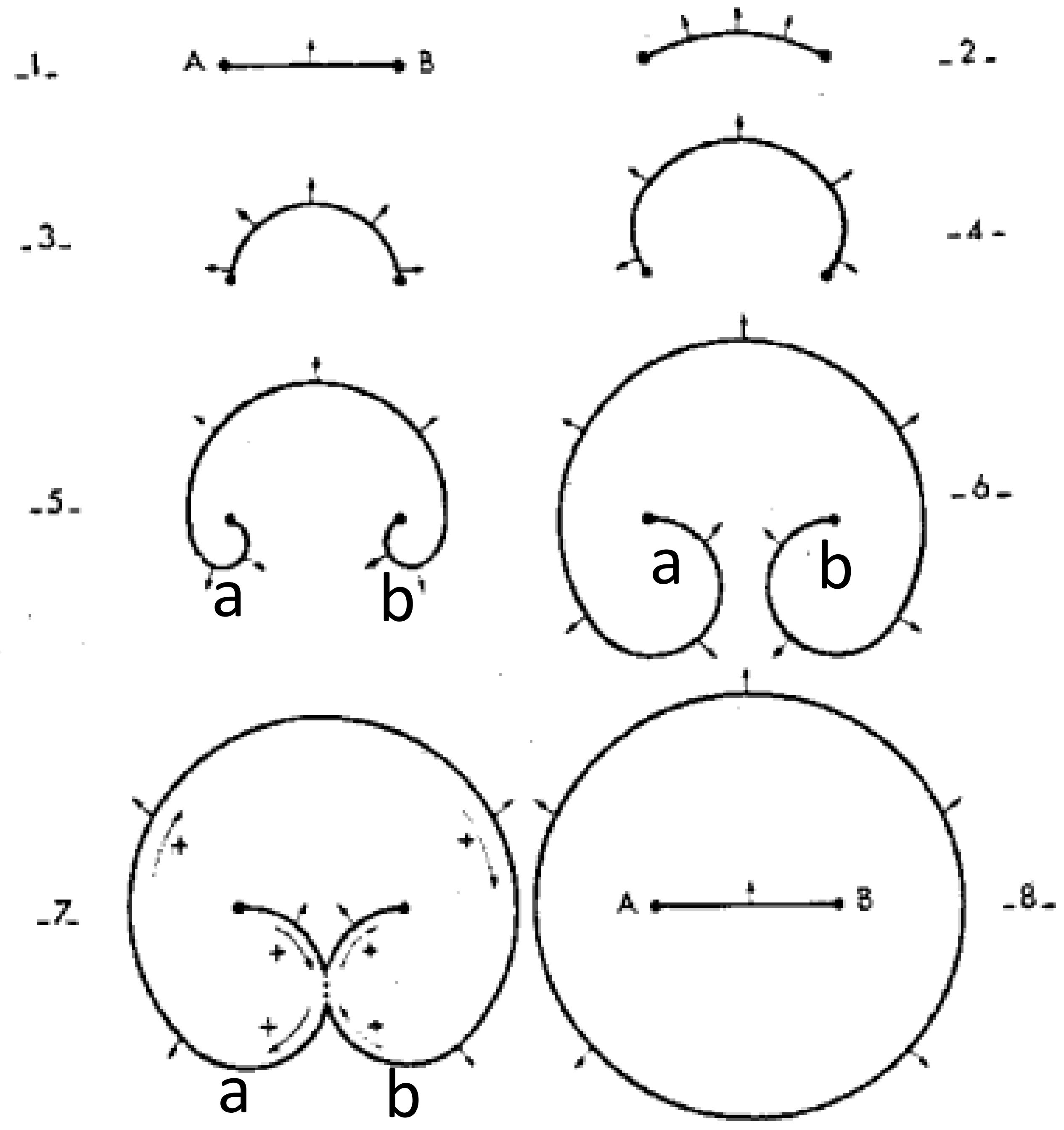


F-R source in Si



Dislocations lie along <110>

$$\tau_{max} = \mu b / L$$



Applied force normal to line

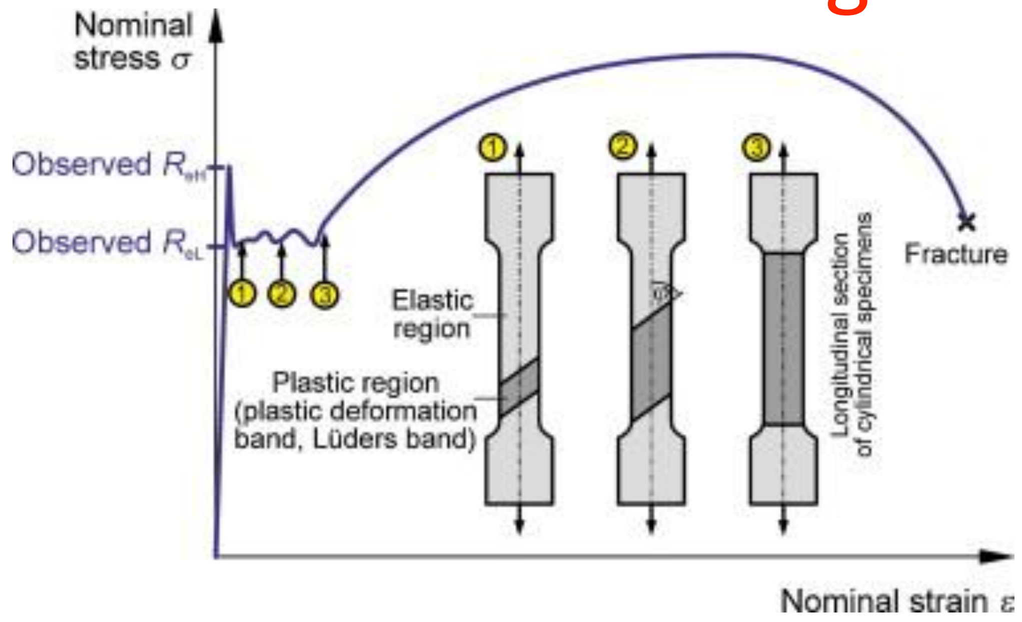
Dislcoation bows

a and b segments curl under tension

a and b line touch. Since they have opposing line vectors, they cancel, leaving pinned line and a loop

Other hardening modes

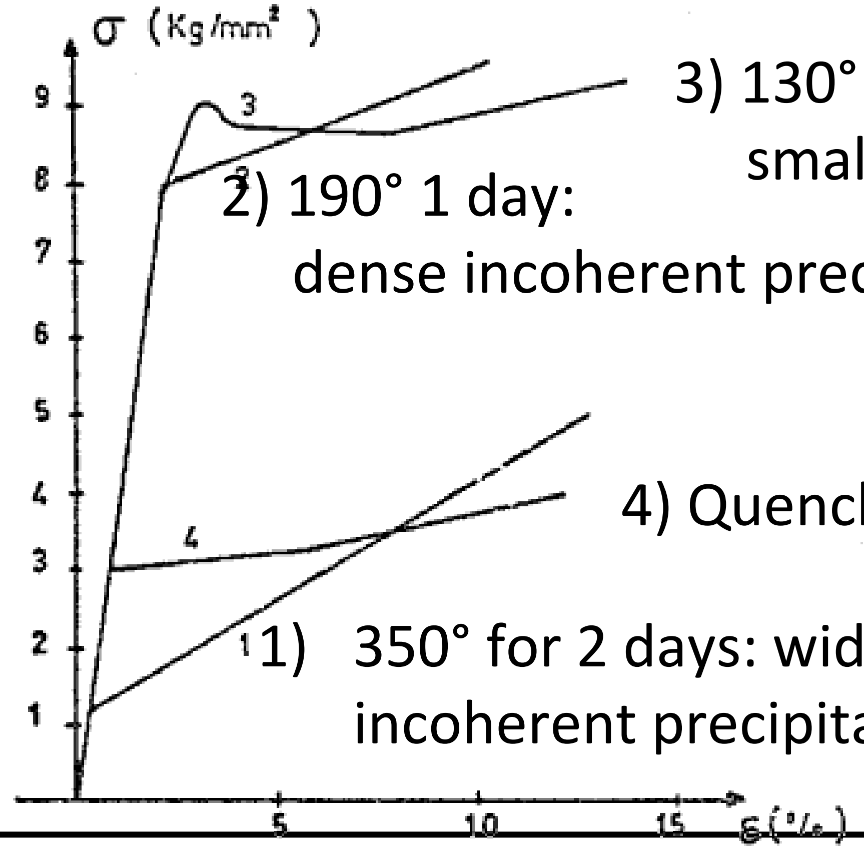
Pinning



Mild Steel to carbon

Structural hardening: precipitates

Al-2%Cu



3) 130° 2 days: dense small coherent precipitates

2) 190° 1 day: dense incoherent precipitates

4) Quenched: solid solution

1) 350° for 2 days: widely spaced incoherent precipitates

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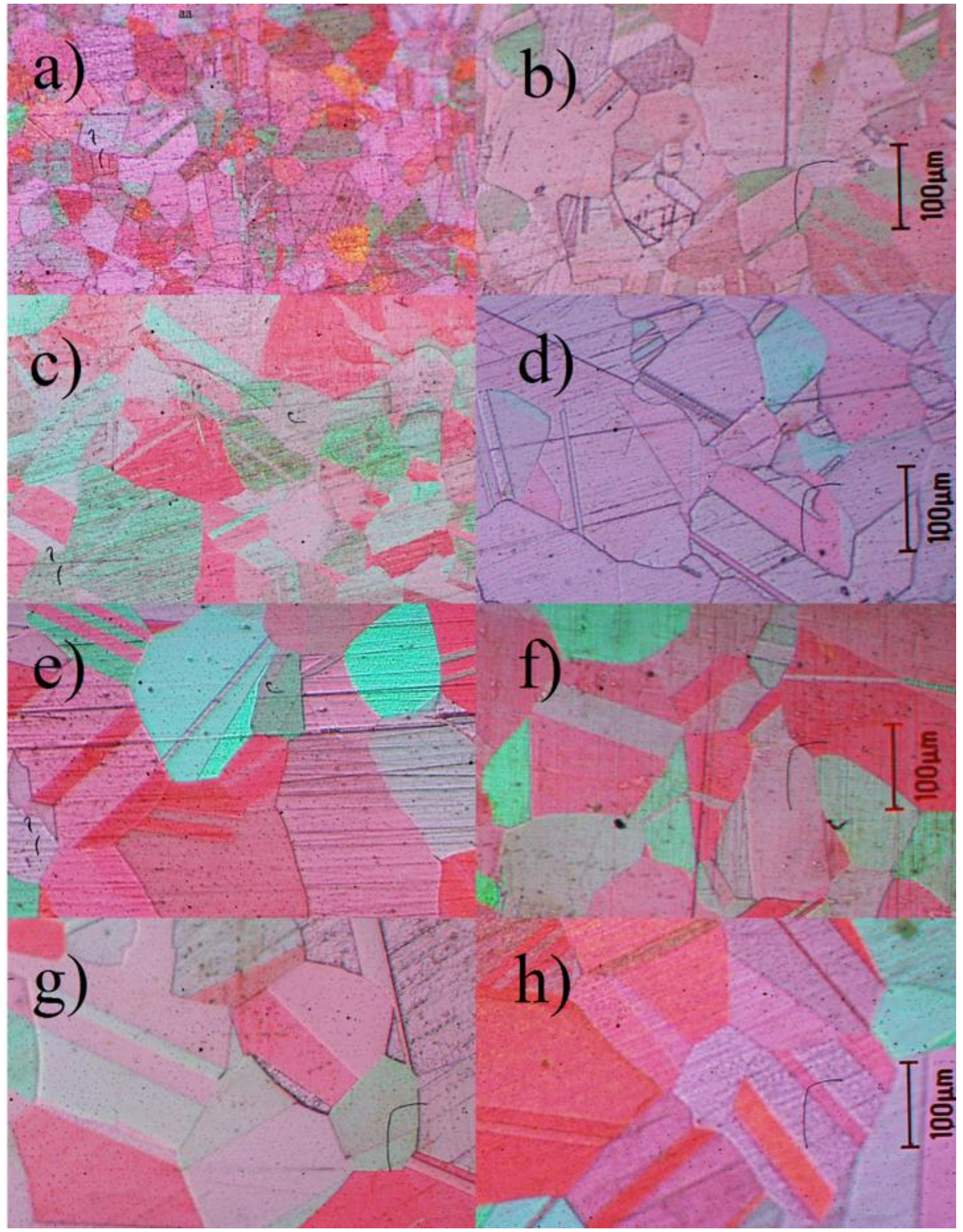
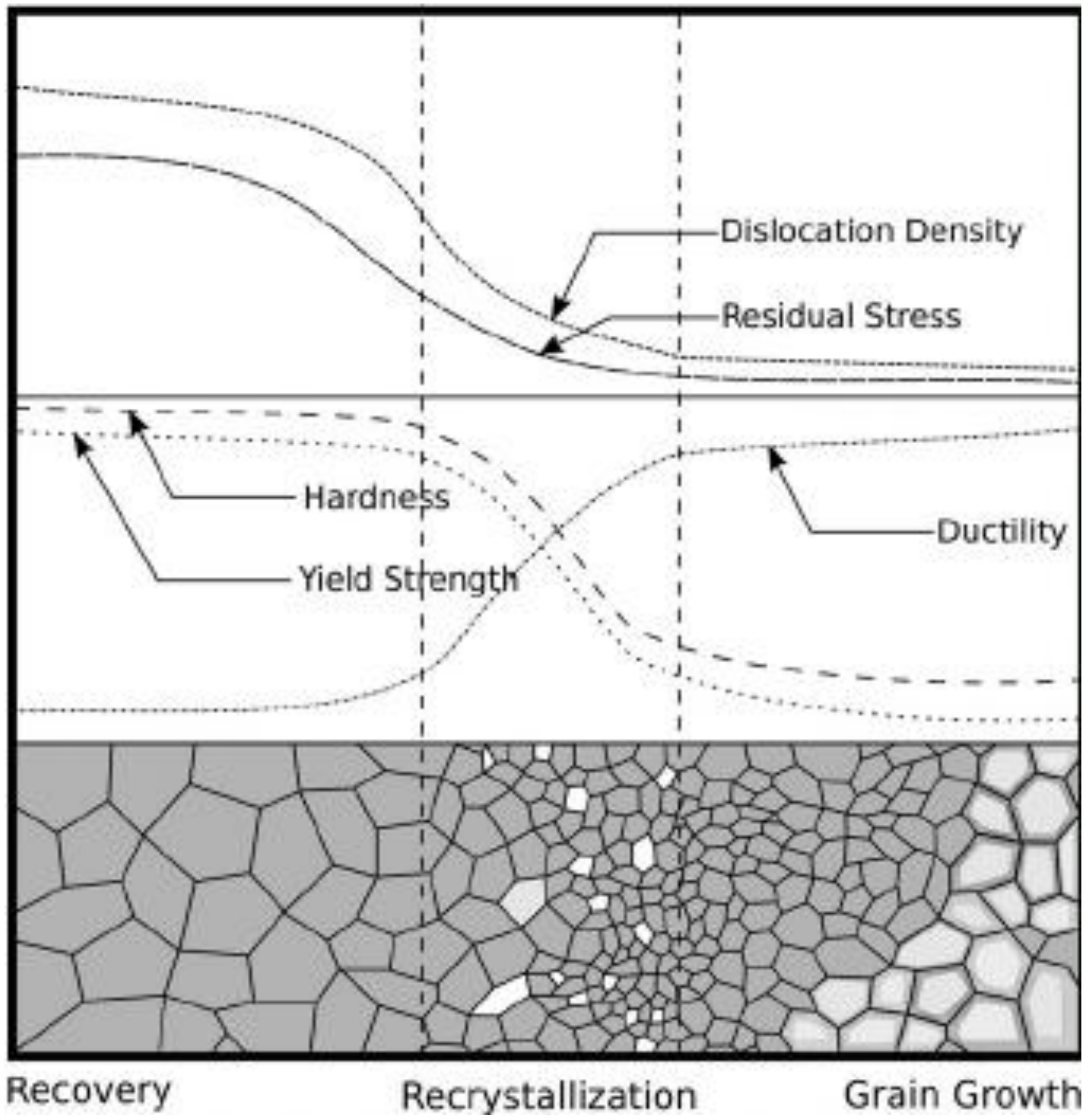
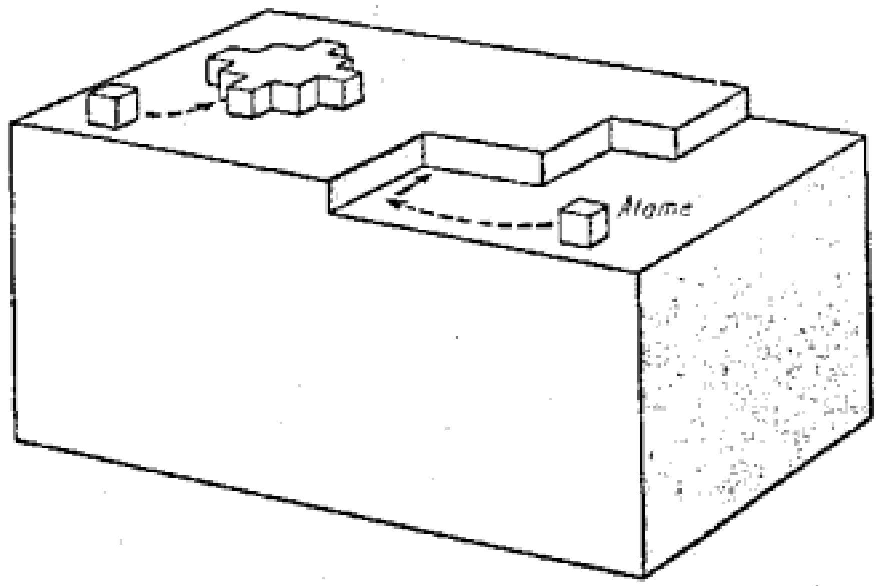
Recrystallization

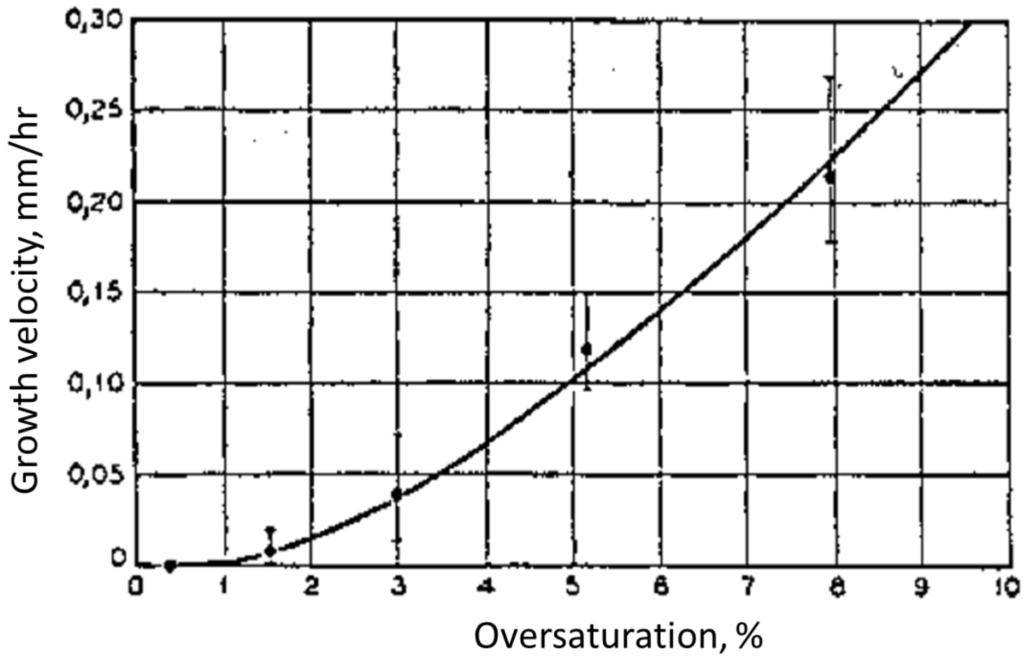




Increase of the grain size in gold

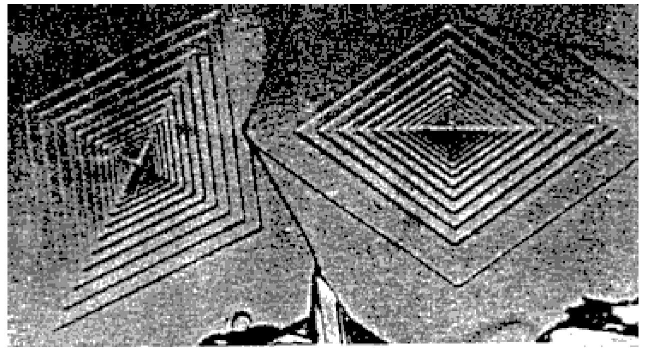
Growth of chiral crystals



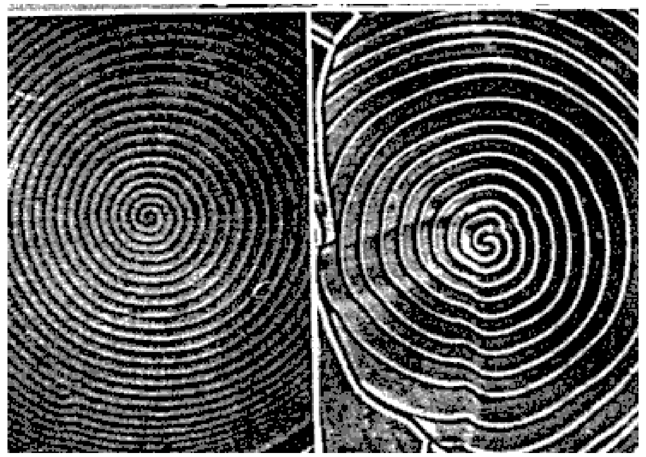


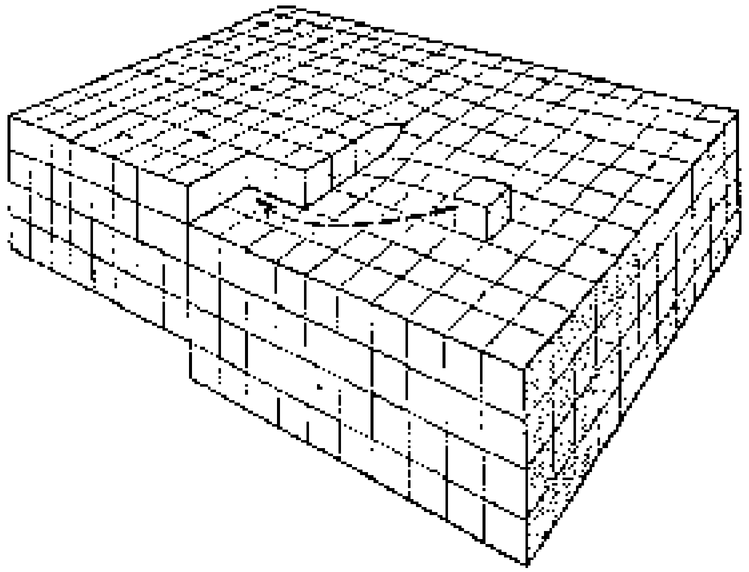
Growth of crystals depending on supersaturation

Polyethylene crystal



SiC





$$\rho = 2\rho_c\theta$$

$$\omega = \frac{d}{dt}\left(\frac{\rho_\infty}{2\rho_c}\right) = \frac{v_\infty}{2\rho_c}$$